

Fiscal Consequences of Missing an Inflation Target

Michele Andreolli¹ and Hélène Rey²

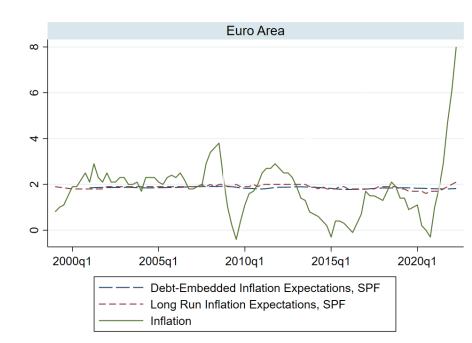
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August 2023

Motivation

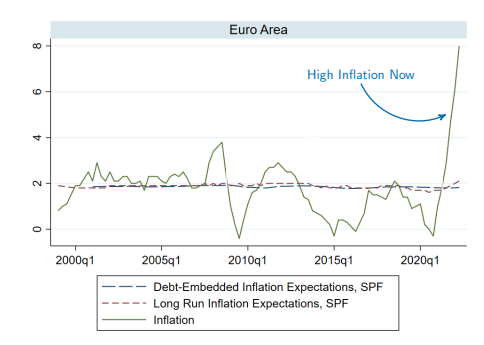
- Does missing an inflation target have an effect on fiscal balances?



Source: ECB Survey of Professional Forecasters (SPF). Sample: 1999Q1 - 2022Q2.

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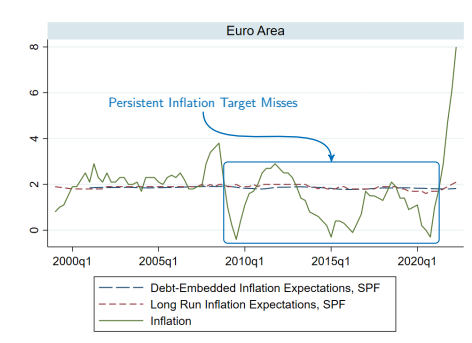
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- High inflation now



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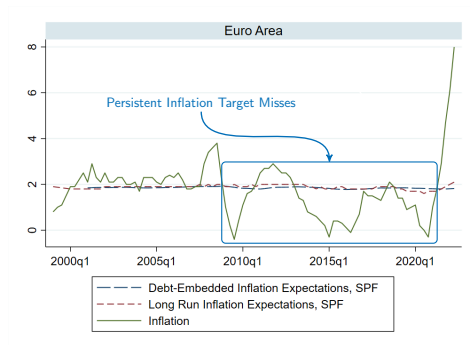
- Does missing an inflation target have an effect on fiscal balances?
- High inflation now, but persistent inflation below target for more than a decade.



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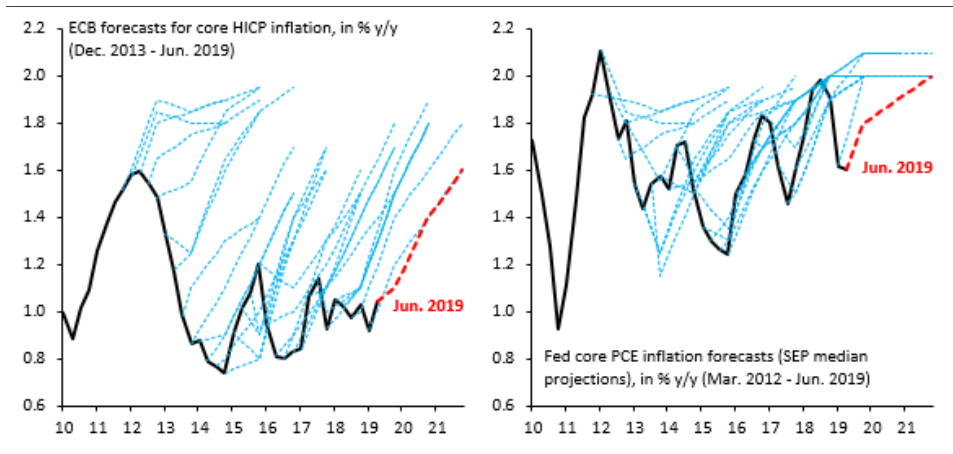
Motivation

- Does missing an inflation target have an effect on fiscal balances?
- High inflation now, but persistent inflation below target for more than a decade.
- Heterogeneous debt level and maturity across Euro Area countries.
- What is the implication for the optimum currency areas literature?



Source: ECB Survey of Professional Forecasters (SPF). Sample: 1999Q1 - 2022Q2.

Central Banks' Inflation Forecast Errors



Sample: 2012Q1 - 2019Q2.

Optimum Currency Area

- Mundell (1961) theory of Optimum Currency Areas had many insights for the Euro.
- Focus mainly on role of trade, factor mobility, monetary policy response to shocks, and international currency role (Rose, 2000; Blanchard and Katz, 1992; Frankel and Rose, 1998; Krugman, 1993; Portes and Rey, 1998).

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- Big central issue: **one monetary policy with heterogeneous fiscal policies.**
 - Monetary-fiscal interaction *ex-ante*: moral hazard, financial regulation
 - Monetary-fiscal interaction *ex-post*: lack of fiscal transfers, heterogeneous transmission of monetary policy

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 - Monetary-fiscal interaction *ex-ante*: moral hazard, financial regulation
 - Monetary-fiscal interaction *ex-post*: lack of fiscal transfers, heterogeneous transmission of monetary policy
- ⇒ **THIS PAPER:** A credible Central Bank persistently missing its target has sizable heterogeneous effects on fiscal accounts. May lead to more divergence across countries.

Our Paper

- Document sizeable and heterogeneous fiscal costs of persistently missing the inflation target for a credible central bank.
- Quantitative Analysis:
 - Size of costs depends on debt maturity, debt levels and inflation expectations.
 - Simple rule of thumb: $\text{fiscal costs} = \text{Duration-to-GDP} \times \text{debt-embedded inflation forecast errors}$.
 - Decomposition of fiscal costs
 - Counterfactual analysis.

Results

- In high debt high maturity countries (e.g. Italy or Belgium) fiscal costs higher by 30% of GDP since inception of the Euro.
- Lower effects for low-maturity low-debt countries.
- Mechanism:
 - Lower inflation leads to higher debt burden on legacy debt (Fisher effect on legacy debt).
 - Long run expectations, anchored at target, affect the cost of newly issued debt.

⇒ **Interaction of long dated debt and mispricing leads to large effects.**

Literature

- Optimal Currency Area: Mundell (1961); Rose (2000); Blanchard and Katz (1992); Kenen (1969); Frankel and Rose (1998); Krugman (1993); Portes and Rey (1998); Begg et al. (1998).
- Public Debt and Inflation: Hall and Sargent (2011); Ellison and Scott (2017); Reinhart and Sbrancia (2015); Hilscher, Raviv and Reis (2021); Blanchard and Leigh (2013); Blanchard (2019); Missale and Blanchard (1994); Acalin and Ball (2022); Chiang and LaBelle (2022)
- Public debt structure and monetary policy: Andreolli (2022); Wolf and Zessner-Spitzenberg (2021); Berger, Dell'Ariccia and Obstfeld (2018); Auerbach and Obstfeld (2005); Reichlin, Ricco and Tarbé (2021); Willems and Zettelmeyer (2022); Bianchi, Melosi and Rogantini Picco (2022)
- Deviation from FIRE: Coibion and Gorodnichenko (2015); Candia, Coibion and Gorodnichenko (2022); Stavrakeva and Tang (2021)
- Measuring Inflation Expectations: Ang, Bekaert and Wei (2007); Pennacchi (1991); Hördahl and Tristani (2018); Haubrich, Pennacchi and Ritchken (2012); Garcia and Werner (2010); Cohen, Hördahl and Xia (2018); Breach, D'Amico and Orphanides (2020); Reis (2020)

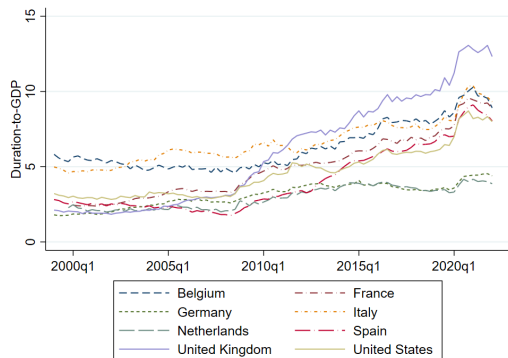
Duration-to-GDP

- DEFINITION Duration-to-GDP: how much the market value of public debt to GDP declines following a one percent increase in interest rates.
- PROPOSITION (Andreolli, 2022): If change is permanent, duration-to-GDP is the NPV of debt servicing costs savings on existing debt compared with overnight debt.

$$DurGDP_t = \frac{\sum_{j=1}^{\infty} j e^{-jr_{t,j}} d_{t,j}}{GDP_t} = MacDur_t * DebtGDP_t$$

Duration-to-GDP

- Build dataset 1999Q1-2022Q1 Duration-to-GDP
- Securities Public Debt from the OECD and Macaulay Duration from WGBI FTSE



Country	All Sample			Most Recent Values		
	DurGDP	MacDur	Debt	DurGDP	MacDur	Debt
France	5.0	6.8	70	8.9	8.8	101
Germany	3.1	6.5	47	4.4	8.1	54
Italy	6.6	6.5	100	9.3	7.3	127
Belgium	6.3	7.2	88	8.9	9.8	91
Spain	4.1	6.4	61	8.0	7.9	102
Netherlands	3.0	6.6	44	3.9	9.1	43
Austria	4.7	7.3	63	7.5	10.3	73
Portugal	3.6	5.7	61	5.5	7.1	78
United States	4.6	5.7	79	8.0	6.8	117
United Kingdom	6.0	10.0	55	12.3	13.3	93

More Countries:

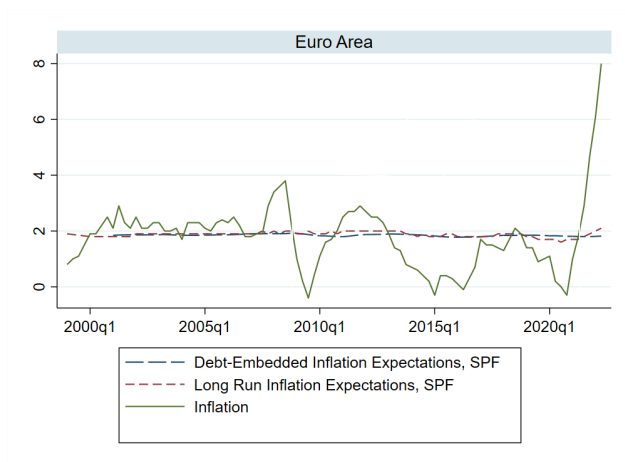
[Table](#)

[Figure](#)

Measuring Long Run Inflation Expectations

- **Debt-Embedded inflation expectations:** $\mathbb{E}_{t-j}(\pi_t|t-j)$ weighted by how much debt outstanding today was issued j periods ago.
- Use Survey of Professional Forecasters (ECB and Fed) for inflation forecasts.
 - Robust to market based metrics as Inflation Linked Swaps. [Details](#)
- Use WGBI FTSE maturity for weighting. [Details](#)

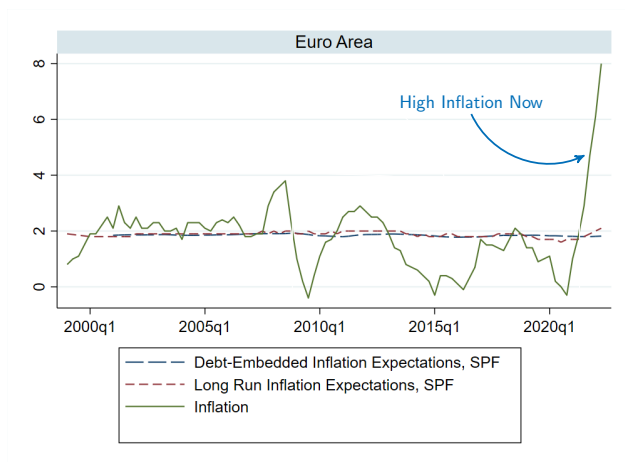
Measuring Long Run Inflation Expectations



- Debt-Embedded Inflation Expectations are very close to Long Run Expectations and to the Inflation Target (1.9%).
- High credibility of the ECB.

Source: ECB Survey of Professional Forecasters (SPF). Sample: 1999Q1 - 2022Q2.

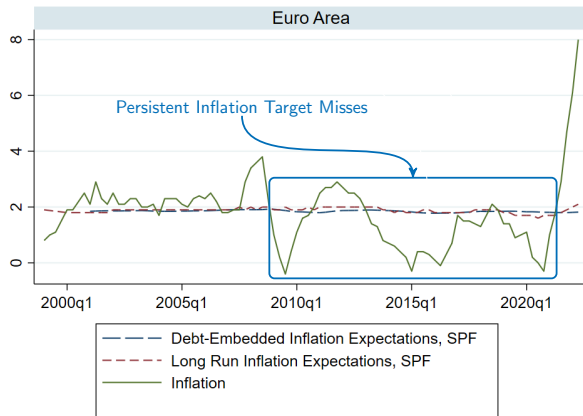
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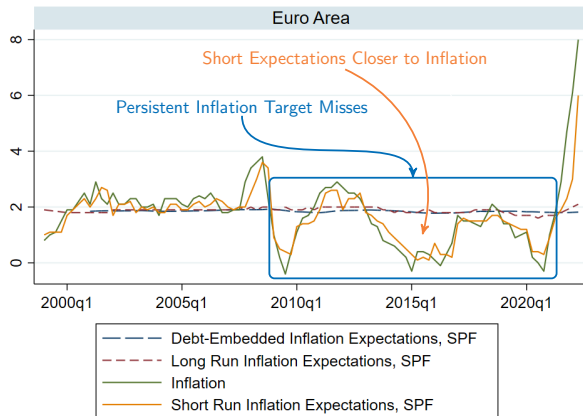
Measuring Long Run Inflation Expectations



- For a large part of the post 2009 period, the ECB (and the Fed) systematically undershot the inflation target. [Plot by Country](#)
- from 2009q1 to 2020q1: [Full Table](#)
 - Inflation was below long run inflation expectations for 75% of the time.
 - Inflation averaged 1.25% vs debt-embedded expectations of 1.84%. (forecast error -0.59 and statistically different from zero).

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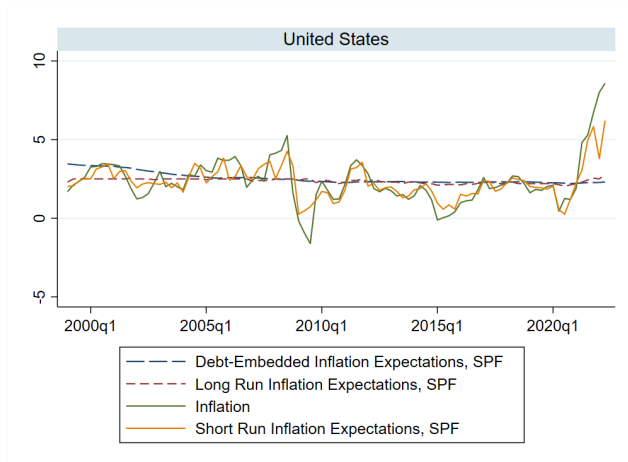
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 - Inflation was below long run inflation expectations for 75% of the time.
 - Inflation averaged 1.25% vs debt-embedded expectations of 1.84%. (forecast error -0.59 and statistically different from zero).
 - Short run expectation forecast errors much closer to zero at -0.04 (not statistically different from zero).

Missing an Inflation Target



- Similar pattern in the US.

Source: Philadelphia Fed Survey of Professional Forecasters (SPF). Sample: 1999Q1 - 2022Q2.

Fiscal Implications of Missing an Inflation Target

- Given public debt amounts, maturity, premia and convenience yields.

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- Given public debt amounts, maturity, premia and convenience yields.
- Persistent undershooting of inflation target with anchored long run inflation expectations has two effects:
 - Lower inflation π increases debt burdens on legacy debt (Fisher effect).
 - Mispricing on long inflation expectation $\mathbb{E}(\pi)$ affects long rates on newly issued debt.

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- Persistent undershooting of inflation target with anchored long run inflation expectations has two effects:
 - Lower inflation π increases debt burdens on legacy debt (Fisher effect).
 - Mispricing on long inflation expectation $\mathbb{E}(\pi)$ affects long rates on newly issued debt.
- A systematic positive mistake on embedded inflation expectations leads to an overestimate of long term nominal rates of the same amount.
- **Fiscal cost = Forecast error on debt-embedded inflation expectations \times Duration-to-GDP:**
 - At each point in time, NPV of debt servicing costs with the current debt profile compared with correctly priced short term debt in GDP units, if the decline in inflation compared to the expectation embedded in debt was permanent.
 - As short term debt is priced using app. correct inflation expectations, we measure at each point in time the fiscal cost of the “mispricing” of longer dated debt.

Fiscal Consequences of Missing Inflation Targets

Country	Cost	
	Mean	SE
France	2.16	(0.61)***
Germany	1.06	(0.36)***
Italy	2.26	(0.76)***
Belgium	2.26	(0.72)***
Spain	2.05	(0.52)***
Netherlands	1.08	(0.34)***
Austria	1.80	(0.55)***
Portugal	1.31	(0.42)***
Ireland	1.04	(0.30)***
Finland	1.01	(0.28)***
United States	2.23	(0.54)***
Average	1.66	(0.16)***

Sample: 2001Q1 - 2021Q1

Figure

- Forecast error on debt-embedded inflation expectations \times Duration-to-GDP.
- Large costs of missing the inflation target!
- 2.26% higher payments per year in Italy and Belgium. High debt level and maturity countries.
- Low for low debt and low maturity countries. Finland or Germany 1%.
- Not a North-South divide: Portugal 1.31% was hit less than Austria 1.8% as lower maturity in Portugal.

Fiscal Consequences of Missing an Inflation Target - Robustness

- Results are robust also with longer sample: Longer Sample
- Results are robust to including the inflation risk premium: ILS
- For the US we explore also alternative inflation expectation metrics:
 - Full term structure of inflation expectations from surveys.
 - Inflation expectations extracted from term structure models.
 - Break even inflation from difference between TIPS and nominal bonds (include also risk premium and convenience yield).
- Results are robust: Alternative Expectations
- Results are robust to excluding ECB Quantitative Easing bond holdings. No ECB QE
- Results are robust to including non-marketable debt. Non Marketable Debt
- Results are robust to segmentation across Euro Area sovereign, by using country specific inflation linked swaps and realised inflation. Segmentation

Computational Exercise Description

- Fit partial equilibrium model to data to perform counterfactual experiments: government budget constraint, public debt law of motion and Euler bond pricing.
- Quantify fiscal costs of missing the inflation target, with a credible Central Bank.
- Hold primary surplus, inflation realization, full term structure inflation expectations, GDP growth, risk premia, convenience yields as in the data.

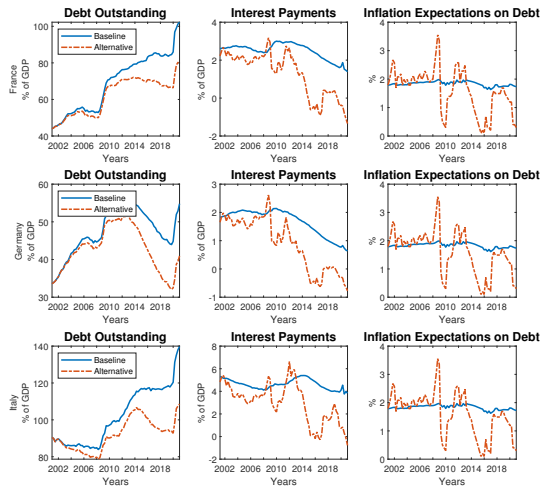
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- Persistent but not permanent decline in inflation. [Theory details](#)

Computational Exercises

1. What if maturity was short?
2. What if we had perfect foresight for inflation expectations up to end of sample ($\mathbb{E}_t(\pi_{t+j}) = \pi_{t+j}$)?
 - See effect on debt at end of sample and interest payments per year.
 - *Ceteris paribus* exercise. [Implementation Details](#)

Computational Exercises



- Blue solid: fit model on data.
- Red dot-dashed: one period debt.
- Debt and interest payments lower under short debt.
- Lower debt-embedded inflation expectations when debt was high under short debt.

Other Countries

Computational Exercises

Country	Short Debt		Perfect Foresight	
	Debt	Interest Payments	Debt	Interest Payments
France	21.4	1.20	3.7	0.21
Germany	13.9	0.78	2.6	0.15
Italy	31.2	1.63	5.6	0.29
Belgium	31.2	1.86	3.7	0.22
Spain	20.6	1.09	3.4	0.18
Netherlands	13.9	0.81	2.0	0.12
Austria	16.7	0.90	3.2	0.18
United States	17.0	1.14	5.1	0.34

Sample: 2001Q1 - 2021Q1

Longer Sample

Alternative Inflation Metric

Experiments Comparison

- Large costs of missing the inflation target!
- 31.2 pp higher debt in Italy and Belgium in data vs short debt.
- 13.9 pp in Germany and Netherlands.
- Lower but still present effect under data vs perfect foresight:
 - 5.6% of GDP in Italy: 100 billion EUR (2021 nominal GDP).
 - 5.1% of GDP in the US: 1.1 trillion USD (2021Q1 nominal GDP).

Computational Exercises Comparison with Duration-to-GDP

- Compare (short-vs-long debt) in computational exercise with empirical (Duration-to-GDP * forecast error).

Country	Counterfactual Exercise	Empirical Result	
	Estimate	Estimate	Confidence Interval
France	1.20	2.16	[0.97 , 3.36]
Germany	0.78	1.06	[0.34 , 1.77]
Italy	1.63	2.26	[0.78 , 3.75]
Belgium	1.86	2.26	[0.86 , 3.67]
Spain	1.09	2.05	[1.03 , 3.07]
Netherlands	0.81	1.08	[0.41 , 1.75]
Austria	0.90	1.80	[0.72 , 2.88]
United States	1.14	2.23	[1.16 , 3.29]

Difference in average interest payments per year. 95% confidence interval.

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Difference in average interest payments per year. 95% confidence interval.

- Compare (short-vs-long debt) in computational exercise with empirical (Duration-to-GDP * forecast error).
- Empirical exercise yields higher numbers compared to the counterfactual exercise \Rightarrow inflation was below target persistently but the miss was not permanent.
- Order of magnitude is similar in the two experiments. See 95% confidence intervals.
- Duration-to-GDP metric is simple but tracks the order of magnitude well: useful benchmark for Central Banks, Debt Management Offices, and IMF.

Conclusion

Large Fiscal Consequences of Missing the Inflation Target

What? Depends on Public Debt Level and Maturity and CB Credibility.
Costs up to 30% of GDP for high debt and high maturity countries.

How? Duration-to-GDP and debt-embedded inflation forecast errors.
Theory Results: effect on legacy debt and newly issued debt.
Computational Exercise: can use Duration-to-GDP as simple metric.

So what? Optimal Currency Area
Distribution.

Thank You!

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Fiscal Implications of Missing an Inflation Target - Why Should We Care?

- Bond holders gained on government bonds portfolio.
- Higher taxation in net present value.
- If Ricardian equivalence holds, irrelevant even with mispricing.
- If it doesn't, mainly rich older households and foreigners gain (see Doepke and Schneider, 2006, in the US).

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- If it doesn't, mainly rich older households and foreigners gain (see Doepke and Schneider, 2006, in the US).
- Corollary of this paper
 - and Doepke and Schneider (2006) younger middle class who hold mortgages lose.
 - and Gourinchas and Rey (2007) (United States has relatively more fixed debt liabilities and more equity and FDI assets): foreigners tend to gain relatively from the misses in the inflation target, holding the exchange rate fixed.
- Persistent inflation target misses in the past decade can be one of the contributors of the increase in inequality post-financial crisis.

Fiscal Implications of Missing an Inflation Target - Why Should We Care?

- For Euro Area, heterogeneous debt levels and maturities have generated heterogeneous fiscal losses across countries for the same monetary policy.
- If fiscal policy directly responds to debt levels (Stability and Growth Pact), the misses may constitute an amplification mechanism and, in some cases lead to austerity, for example.
- Leads to heterogeneous transmission of monetary policy, if public debt level and maturity affect its strength (Andreolli, 2022).

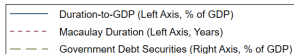
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Austria	4.7	7.3	63	7.5	10.3	73
Belgium	6.3	7.2	88	8.9	9.8	91
Czech Republic	2.1	5.9	27	2.5	6.5	39
Denmark	2.3	6.9	34	2.5	9.6	26
Finland	2.4	5.7	40	4.2	8.2	52
France	5.0	6.8	70	8.9	8.8	101
Germany	3.1	6.5	47	4.4	8.1	54
Hungary	2.8	4.4	56	3.6	5.3	67
Ireland	2.5	6.5	39	3.3	9.0	36
Italy	6.6	6.5	100	9.3	7.3	127
Netherlands	3.0	6.6	44	3.9	9.1	43
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Spain	4.1	6.4	61	8.0	7.9	102
Sweden	1.8	5.7	33	1.3	6.0	21
United Kingdom	6.0	10.0	55	12.3	13.3	93
United States	4.6	5.7	79	8.0	6.8	117

Notes: This table shows Duration-to-GDP in the second and fifth columns, Macaulay duration in the third and sixth columns, and public debt securities in the fourth and seventh columns. Macaulay duration source is the FTSE WGBI data and public debt securities is the OECD. Columns 2 to 4 show the average value on the 1999Q1 to 2022Q1 sample or longest available sample, whereas columns 5 to 7 show the most recent value: 2022Q1.

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Duration-to-GDP for all countries



Notes: The time series presents Duration-to-GDP (in the blue solid line), Macaulay duration (in the red short dashed line), and government debt securities (in the green long dashed line) for all the countries in the dataset. Macaulay duration data is FTSE WGBI. Debt data are government debt securities over GDP from the OECD. The sample goes from 1999Q1 to 2022Q1.

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Measuring Long Run Inflation Expectations

- Surveys: e.g. Survey of Professional Forecasters.
- Market based metrics: e.g. 5-Year, 5-Year Forward Inflation Rate measured with Inflation Linked Swaps.

Measuring Long Run Inflation Expectations

- Surveys: e.g. Survey of Professional Forecasters.
 - Pros (1): direct measures of inflation expectations.
 - Pros (2): forecast inflation better than asset markets and macro variables (Ang, Bekaert and Wei, 2007).
 - Pros (3): term structure models use them to improve fit and forecasting ability.
 - Cons: can never be sure of the truthfulness of survey responses.
- Market based metrics: e.g. 5-Year, 5-Year Forward Inflation Rate measured with Inflation Linked Swaps.
 - Cons (1): embed risk premia, liquidity premia, convenience yields.
 - Cons (2): forecast inflation worse than surveys.
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 - Pros (1): actual transactions.
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Measuring Long Run Inflation Expectations

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- **Market based metrics. e.g. 5-Year, 5-Year Forward Inflation Rate measured with Inflation Linked Swaps.** ← **but results robust to these.**
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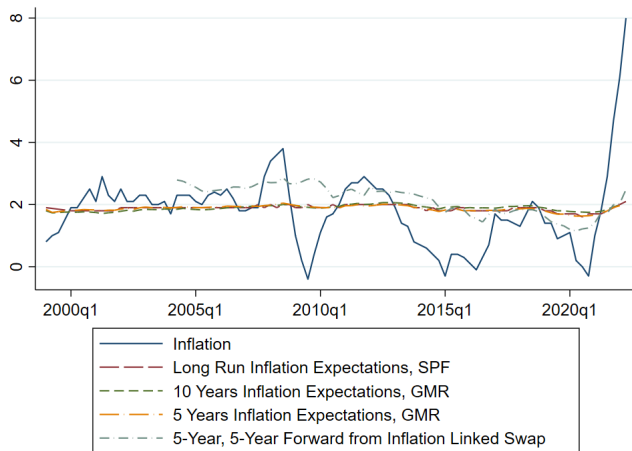
Debt-Embedded Inflation Expectations

- For Debt-Embedded Inflation Expectations we take a issuance weighted average of inflation rates:

$$\sum_{j=1}^{J-1} \mathbb{E}_{t-j}(\pi_{t|t-j}^a) \frac{1}{Mat_t} \left(1 - \frac{1}{Mat_t}\right)^{j-1} + \mathbb{E}_{t-J}(\pi_{t|t-J}^a) \left(1 - \frac{1}{Mat_t}\right)^J$$

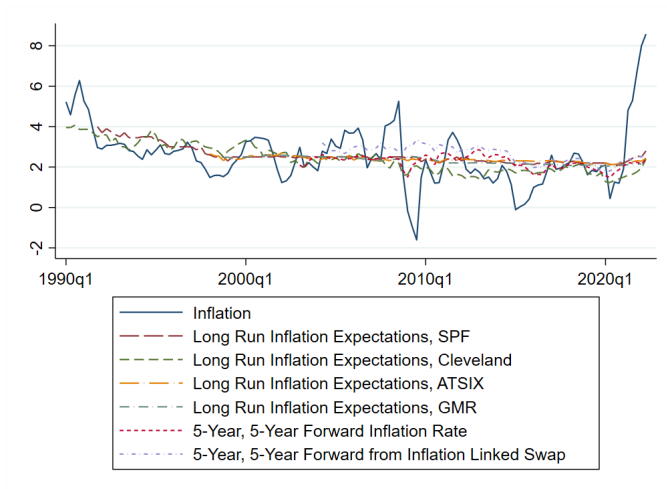
- $\pi_{t|t-j}^a$ is the annualized inflation rate from period $t - j$ to t .
- Mat_t is the maturity of public debt: average life from the WGBI FTSE data.
- Formula from the theoretical framework, with a constant fraction of debt being repaid each period.
- Use SPF data: for $\mathbb{E}_{t-1}(\pi_{t|t-1}^a)$ one year head we use inflation expectations, for $\mathbb{E}_{t-2}(\pi_{t|t-2}^a)$ we use two years head inflation expectations, and for any $\mathbb{E}_{t-j}(\pi_{t|t-j}^a)$ with $j > 2$ we use the long run inflation expectation.
- As robustness, we also employ full term structure of inflation expectations by Grishchenko, Mouabbi and Renne (2019), Aruoba (2020), and Haubrich, Pennacchi and Ritchken (2012). All very close.

Long Run Inflation Expectations Comparison in the Euro Area



Notes: This graph presents long run inflation expectation measures and realised inflation for the Euro Area. The SPF series is the 4/5 years ahead YoY inflation from the ECB Survey of Professional Forecasters (SPF). 10 (5) Years GMR is the 10 (5) years ahead inflation expectation from the Grishchenko, Mouabbi and Renne (2019) measure. Inflation and its expectations pertain to the Harmonised Index of Consumer Prices (HICP). The sample goes from 1999Q1 to 2022Q2. The 5 year by 5 year forward inflation linked swap sample starts in 2012Q4. [Back](#)

Long Run Inflation Expectations Comparison in the United States



Notes: This graph presents long run inflation expectation measures and YoY CPI realised inflation for the United States. The SPF series is the 10 years average inflation from the Philadelphia Fed Survey of Professional Forecasters (SPF). Cleveland is the Cleveland FED 10 year ahead expected inflation, ATSIX is the mean inflation expectation 10 years ahead from Aruoba (2020). GMR is the 10 years ahead inflation expectation from the Grishchenko, Mouabbi and Renne (2019) measure. 5-Year, 5-Year Forward Inflation Rate is measured with nominal and inflation linked treasury rates. The sample goes from 1990q1 to 2022Q2. The 5 year by 5 year forward inflation linked swap sample starts in 2012Q4.

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SPF Inflation Expectations Comparison and Inflation

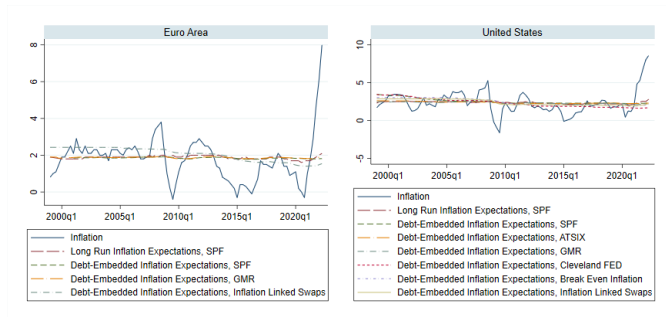


Notes: This graph presents all the Survey of Professional Forecasters (SPF) inflation expectations across horizons for the Euro Area (inflation is HCPI) in the first panel and for the United States (inflation is CPI) in the second panel.

The sample goes from 1999q1 to 2022Q2.

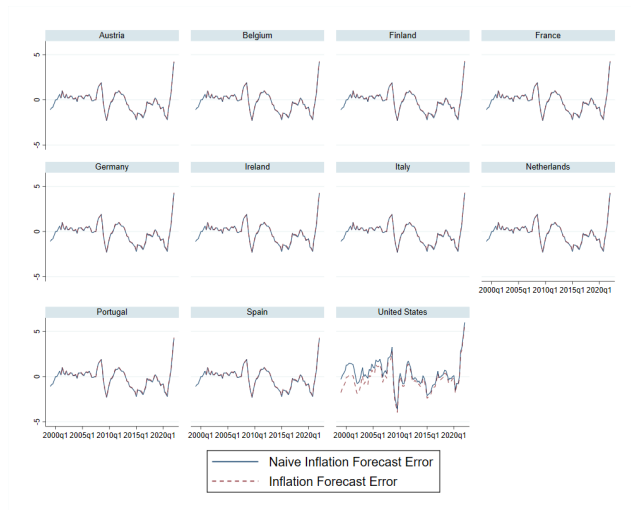
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Inflation Expectations on Debt Comparison and Inflation in the Euro Area and in the United States



Notes: This graph presents debt-embedded inflation expectations and YoY inflation for the Euro Area (HCPI) and for the United States (CPI). The long run inflation expectations are the Survey of Professional Forecasters (SPF) series, the 4/5 years ahead YoY inflation from the ECB for the Euro Area and the 10 years average inflation from the Philadelphia Fed for the United States. Cleveland is the Cleveland FED model, ATSIX is the mean inflation expectation from Aruoba (2020). GMR is the inflation expectation from the Grishchenko, Mouabbi and Renne (2019) measure. The sample goes from 1990q1 to 2022Q1. [Back](#)

Inflation Forecasts Errors for Euro Area Countries and the United States



Notes: This graph presents the forecast error on debt-embedded inflation expectations. Inflation expectations come from the ECB (for the Euro Area on HICP inflation) and Philadelphia Fed (for the US on Headline CPI) Surveys of Professional Forecasters (SPF). The naive approach assumes a inflation expectation of 1.9% for the Euro Area and 2% for the US. The vertical axis units are percentage points. The sample goes from 1999Q1 to 2022Q2. [Back](#)

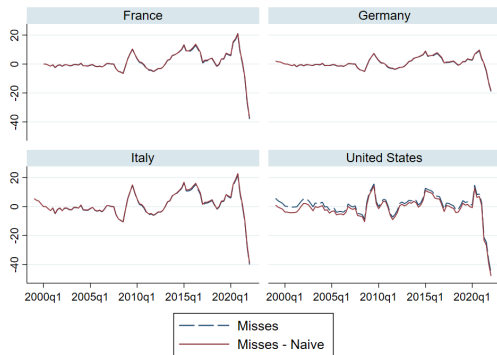
Inflation Forecasts Errors

Sample	Forecast	Euro Area		United States	
		Mean	SE	Mean	SE
From 2001q1 to 2020q1	On Debt	-0.15	(0.10)	-0.38	(0.13)
	Naive	-0.20	(0.10)	0.11	(0.14)
	Short Run	0.03	(0.07)	-0.07	(0.10)
From 2009q1 to 2020q1	On Debt	-0.59	(0.13)	-0.71	(0.16)
	Naive	-0.65	(0.13)	-0.41	(0.16)
	Short Run	-0.06	(0.09)	-0.18	(0.13)
From 2001q1 to 2022q2	On Debt	-0.04	(0.14)	-0.13	(0.17)
	Naive	-0.09	(0.14)	0.33	(0.17)
	Short Run	0.17	(0.10)	0.10	(0.12)

Notes: This table shows the average inflation forecast errors in the Euro Area and in the United States. The first columns shows the sample under study. The second column shows the inflation forecast choice: "On Debt" is the debt-embedded inflation forecast, "Naive" is an inflation forecast of 1.9% in the Euro Area and 2.0% in the United States, and "Short Run" is the lagged (one quarter) forecast for current year inflation. All forecasts are based on the Survey of Professional Forecasters. The forecast error is the current inflation minus the inflation forecast. The third and fifth column show the average inflation forecast error in percentage points. The fourth and sixth columns show the standard error of the average and are obtained by running a regression of the inflation forecast error variable on a constant with White robust errors.

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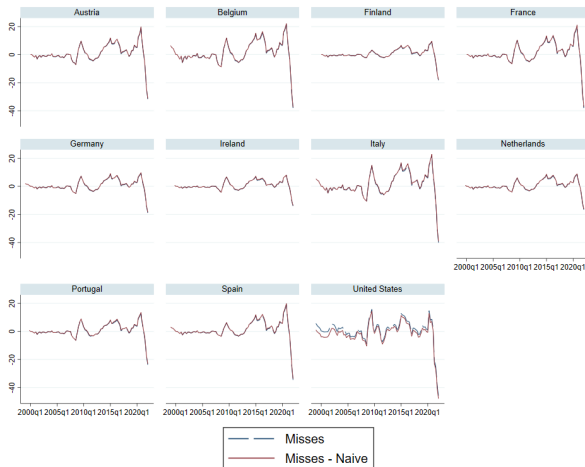
Fiscal Consequences of Missing Inflation Targets



- Forecast error on debt-embedded inflation expectations \times Duration-to-GDP.
- Small misses (pre 2008) happened when debt and maturity were low.
- Big misses happened later.
- Average over sample.

[All Countries](#)[Back](#)

Fiscal Consequences of Missing Inflation Targets for All Euro Area Countries and for the United States



Notes: This figure shows the fiscal costs of missing the inflation targets on public debt. The “Misses” lines show the multiplication between Duration-to-GDP and the forecast error on debt-embedded inflation expectations with SPF, OECD, and FTSE WGBI data. The “Misses - Naive” show the same multiplication, but with the naive inflation forecast. The vertical axis units are percent of GDP. A positive number implies that inflation is below its forecast and a negative number that inflation is above its forecast. The sample goes from 1999Q1 to 2022Q1.

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Fiscal Consequences of Missing Inflation Targets - With the Recent Inflation Increase

Country	Cost		Naive Cost	
	Mean	SE	Mean	SE
France	1.18	(0.83)	1.47	(0.84)*
Germany	0.57	(0.46)	0.76	(0.46)
Italy	1.23	(0.95)	1.61	(0.96)*
Belgium	1.27	(0.91)	1.56	(0.92)*
Spain	1.15	(0.73)	1.42	(0.74)*
Netherlands	0.64	(0.42)	0.82	(0.42)*
Austria	0.98	(0.73)	1.20	(0.73)
Portugal	0.70	(0.55)	0.94	(0.55)*
Ireland	0.66	(0.36)*	0.82	(0.37)**
Finland	0.54	(0.39)	0.71	(0.40)*
United States	0.61	(0.98)	-1.28	(0.98)
Average	0.87	(0.21)***	0.91	(0.21)***

Notes: This table shows the fiscal costs missing the inflation targets on public debt. The second and third column show the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with SPF, OECD, and FTSE WGBI data. The fourth and fifth columns show the same multiplication, but with the naive inflation forecast. The second and fourth column show the average cost in GDP units. The third and fifth columns show the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. Each row shows a different country, except the last row which shows the simple unweighted average across all countries. The sample goes from 2001Q1 to 2022Q1. Legend:

* $p < .1$; ** $p < .05$; *** $p < .01$.

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Fiscal Consequences. Robustness: Inflation Risk Premium

Country	Baseline		ILS	
	Mean	SE	Mean	SE
France	2.16	(0.61)***	2.60	(0.51)***
Germany	1.06	(0.36)***	1.46	(0.31)***
Italy	2.26	(0.76)***	3.20	(0.63)***
Belgium	2.26	(0.72)***	3.22	(0.59)***
Spain	2.05	(0.52)***	2.20	(0.43)***
Netherlands	1.08	(0.34)***	1.44	(0.29)***
Austria	1.80	(0.55)***	2.45	(0.47)***
Portugal	1.31	(0.42)***	1.60	(0.35)***
Ireland	1.04	(0.30)***	1.24	(0.26)***
Finland	1.01	(0.28)***	1.12	(0.23)***
United States	2.23	(0.54)***	1.55	(0.51)**
Average	1.66	(0.16)***	2.01	(0.13)***

Sample: 2001Q1 - 2021Q1

- Counterfactual clear with risk neutrality: $\mathbb{E}(\pi)$ vs π
- Inflation risk premium: reflects volatility in realized inflation, add premium on nominal long rates:
 - $Var(\pi)$: premium \uparrow .
 - $Cov(\pi, MU)$ at business cycle frequency: if < 0 (demand driven business cycles) premium \downarrow .
 - $Cov(\pi, MU)$ at low frequency: if > 0 (e.g. disaster risk: in war low consumption and high inflation) premium \uparrow .
- Use Inflation Linked Swaps.
- In our sample premium was positive. Results are robust. [Back](#)

Fiscal Consequences of Missing Inflation Targets - Metrics Comparison for the United States

Method	Cost	
	Mean	SE
Baseline	2.23	(0.54)***
Naive	0.35	(0.55)
ATSIX	1.82	(0.55)***
GMR	1.70	(0.54)***
Cleveland Fed	1.01	(0.51)*
Inflation Linked Swaps	1.55	(0.51)***
Break Even Inflation Rate	1.70	(0.51)***

Notes: This table shows the fiscal costs missing the inflation targets on public debt for the United States. It shows the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with different methodologies to compute inflation expectations. Duration-to-GDP is computed with OECD, and FTSE WGBI data. The first row uses SPF data for inflation expectations, the second uses naive (2%) expectations, the third uses inflation expectations coming from Aruoba (2020), the fifth from the Cleveland Fed model, the sixth from Grishchenko, Mouabbi and Renne (2019), and finally the last row uses break even inflation rates as inflation expectations. The second and fourth column shows the average cost in GDP units. The third column shows the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. The sample goes from 2001Q1 to 2021Q1. Legend: * $p < .1$; ** $p < .05$; *** $p < .01$.

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Fiscal Consequences of Missing Inflation Targets - No ECB QE

Country	Baseline		No ECB QE	
	Mean	SE	Mean	SE
France	2.16	(0.61)***	1.84	(0.55)**
Germany	1.06	(0.36)***	0.81	(0.33)**
Italy	2.26	(0.76)***	1.94	(0.70)**
Belgium	2.26	(0.72)***	1.92	(0.66)**
Spain	2.05	(0.52)***	1.68	(0.44)***
Netherlands	1.08	(0.34)***	0.83	(0.31)**
Austria	1.80	(0.55)***	1.45	(0.49)**
Portugal	1.31	(0.42)***	1.02	(0.37)**
Ireland	1.04	(0.30)***	0.89	(0.28)**
Finland	1.01	(0.28)***	0.79	(0.24)**
Average	1.66	(0.16)***	1.32	(0.15)***

Notes: This table shows the fiscal costs missing the inflation targets on public debt with different metrics. It shows the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with different methodologies to compute inflation expectations and inflation realisations. We use SPF data for inflation expectations. In columns 1 and 2 we construct Duration-to-GDP is computed with OECD securities debt data without netting out EBC QE purchases, and FTSE WGBI data. In columns 4 and 5, Duration-to-GDP is computed with OECD securities debt data but where we subtract EBC QE purchases in the Public Sector Purchase Programme (PSPP) and in the Pandemic Emergency Purchase Programme (PEPP). Columns 2 and 4 show the average cost in GDP units. Columns 3 and 5 show the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. The sample goes from 2001Q1 to

2021Q1. Legend: * $p < .1$; ** $p < .05$; *** $p < .01$.

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Fiscal Consequences of Missing Inflation Targets - Also Non-Marketable Debt

Country	Baseline		Also Non-Marketable Loans	
	Mean	SE	Mean	SE
France	2.16	(0.61)***	2.45	(0.70)***
Germany	1.06	(0.36)***	1.38	(0.51)**
Italy	2.26	(0.76)***	2.48	(0.83)**
Belgium	2.26	(0.72)***	2.79	(0.86)**
Spain	2.05	(0.52)***	2.44	(0.62)***
Netherlands	1.08	(0.34)***	1.37	(0.43)**
Austria	1.80	(0.55)***	2.13	(0.66)**
Portugal	1.31	(0.42)***	2.21	(0.62)***
Ireland	1.04	(0.30)***	1.48	(0.40)***
Finland	1.01	(0.28)***	1.44	(0.39)***
United States	2.23	(0.54)***	2.23	(0.54)***
Average	1.66	(0.16)***	2.04	(0.19)***

Notes: This table shows the fiscal costs missing the inflation targets on public debt with different metrics. It shows the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance. We use SPF data for inflation expectations. In columns 1 and 2 we construct Duration-to-GDP is computed with OECD securities debt and FTSE WGBI data. In columns 4 and 5, Duration-to-GDP is computed with OECD debt data on securities and non-marketable loan. Columns 2 and 4 show the average cost in GDP units. Columns 3 and 5 show the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. The sample goes from 2001Q1 to 2021Q1. Legend: * $p < .1$; ** $p < .05$; *** $p < .01$.

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Fiscal Consequences of Missing Inflation Targets - Comparison with Market Segmentation Case

Country	Baseline		ILS		ILS CS	
	Mean	SE	Mean	SE	Mean	SE
France	2.16	(0.61)***	2.60	(0.51)***	2.67	(0.49)***
Germany	1.06	(0.36)***	1.46	(0.31)***	1.49	(0.30)***
Italy	2.26	(0.76)***	3.20	(0.63)***	2.72	(0.73)***
Spain	2.05	(0.52)***	2.20	(0.43)***	3.41	(0.70)***
Average	1.88	(0.29)***	2.36	(0.24)***	2.57	(0.29)***

Notes: This table shows the fiscal costs missing the inflation targets on public debt with different metrics. It shows the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with different methodologies to compute inflation expectations and inflation realisations. Duration-to-GDP is computed with OECD, and FTSE WGBI data. The second and third columns use SPF data for inflation expectations and Euro Area wide HICP inflation realisation, the fourth and fifth columns use directly inflation linked swaps (ILS) on Euro Area wide HICP inflation and Euro Area wide HICP inflation realisation, and finally the sixth and seventh columns use directly inflation linked swaps (ILS) on country specific CPI inflation and country specific HICP inflation realisation. Columns 2, 4, and 6 shows the average cost in GDP units. Columns 3, 5, and 7 show the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. The sample goes from 2001Q1 to 2021Q1. Legend:

* $p < .1$; ** $p < .05$; *** $p < .01$.

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Theoretical Framework

- Partial equilibrium model.
- Government issues long maturity nominal debt, repays δ^d fraction of the principal each period.
- Investors price government bond with risk-neutral utility.

$$d_t = (1 - \delta^d) \frac{1}{g_t \pi_t} d_{t-1} + l_t$$

$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) R_{t-1}^{ave} + \frac{l_t}{d_t} R_t^{new}$$

$$s_t + l_t = (R_{t-1}^{ave} + \delta^d) \frac{1}{g_t \pi_t} d_{t-1}$$

$$\frac{1}{(\delta^d + R_t^{new})} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\frac{1}{\pi_{t+j|t}} \beta^j (1 - \delta^d)^{j-1} \right]$$

Theoretical Framework

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$$d_t = (1 - \delta^d) \frac{1}{g_t \pi_t} d_{t-1} + l_t \Leftarrow \text{Debt Law of Motion (LoM)}$$

$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) R_{t-1}^{ave} + \frac{l_t}{d_t} R_t^{new} \Leftarrow \text{Avg Rate on Debt LoM}$$

$$s_t + l_t = (R_{t-1}^{ave} + \delta^d) \frac{1}{g_t \pi_t} d_{t-1} \Leftarrow \text{Govt Budget Constraint}$$

$$\frac{1}{(\delta^d + R_t^{new})} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\frac{1}{\pi_{t+j|t}} \beta^j (1 - \delta^d)^{j-1} \right] \Leftarrow \text{Bond Euler}$$

- Exogenous: s_t surplus, g_t growth, π_t inflation realization, and $\mathbb{E}_t(1/\pi_{t+j|t})$ inflation expectations.
- Endogenous: debt d_t , issuance l_t , R_t^{new} interest rate on newly issued debt, and average interest rate R_t^{ave} .
- 2 state variables: d_t and R_t^{ave} .
- d_t, l_t, s_t in % of GDP. Derivation

Theory Experiment

- Permanent deflationary shock from $t + 1$ announced in t . π_{t+1} onward.
- Gets incorporated in short run inflation expectations $\mathbb{E}_t(\pi_{t+1})$,
- But not in long run ones $\mathbb{E}_t(\pi_{t+j})$, $j > 1$. Stay anchored to the inflation target.
- Debt structure matters:
 1. Legacy debt has already been priced before any variation in inflation and inflation expectations
 2. Long run expectations, which do not adjust, also affect the cost of newly issued debt.

Theory Experiment

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- Debt structure matters:
 1. Legacy debt has already been priced before any variation in inflation and inflation expectations
 2. Long run expectations, which do not adjust, also affect the cost of newly issued debt.
- Government issues new debt only to refinance the debt coming due.
- Deviation from steady growth path.
- Transversality condition on debt holds: $1 + R > \pi g$.
- Study effect on fiscal burden.

Theory Experiment Results

	Long run expectations don't adjust $\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}$	Long run expectations adjust $\{\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}$
Any Maturity δ^d	$-DurGDP \frac{1-\delta^d}{\pi} \frac{R+\delta^d}{R}$	$-DurGDP \frac{1-\delta^d}{\pi}$
Short Debt $\delta^d = 1$	0	0

- No effect under short debt: correctly priced and expected low inflation.
- Standard reasoning in macro models: missing inflation targets can have effects on central bank credibility but is second order for fiscal matters.
- Need long debt and misspricing. [Derivation](#)

Theory Experiment Results

	Long run expectations don't adjust $\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}$	Long run expectations adjust $\{\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}$
Any Maturity δ^d Short Debt $\delta^d = 1$	$\boxed{-DurGDP} \frac{1-\delta^d}{\pi} \frac{R+\delta^d}{R}$ 0	$\boxed{-DurGDP} \frac{1-\delta^d}{\pi}$ 0

- No effect under short debt: correctly priced and expected low inflation.
- If long run expectations adjust, effect only on legacy debt (DurGDP alone).

$\Rightarrow \frac{1-\delta^d}{\pi} \simeq 1$ with long debt.

Theory Experiment Results

	Long run expectations don't adjust $\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}$	Long run expectations adjust $\{\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}$
Any Maturity δ^d	$-DurGDP \frac{1-\delta^d}{\pi} \frac{R+\delta^d}{R}$	$-DurGDP \frac{1-\delta^d}{\pi}$
Short Debt $\delta^d = 1$	0	0

- No effect under short debt: correctly priced and expected low inflation.
- If long run expectations adjust, effect only on legacy debt (DurGDP alone).
 $\Rightarrow \frac{1-\delta^d}{\pi} \simeq 1$ with long debt.
- Multiplicative factor due to mispricing on newly issued debt can be large (~ 3).
- Need long debt and misspricing. [Derivation](#)
- What if Central Bank pegs interest rates: [Go](#)

Theoretical Framework Details 1

- Public debt is issued via a bond with a geometrically decaying amortization. That bond pays a fixed net nominal interest rate R_t^{new} on new issuances. The principal due decays at rate δ^d .
- Each period, the government issues L_t nominal bonds
- End of period stock of debt in each period \rightarrow sum of remaining past issuances
- Recursive formulation for debt D_t and average interest rate R_t^{ave} process on the debt stock. Defined as end of period variables.
- Define F_t to be the debt payments on bonds in period t .
- Debt dynamics system: [Back](#)

$$D_t = L_t + (1 - \delta^d)D_{t-1}$$

$$R_t^{ave} = R_t^{new} \frac{L_t}{D_t} + R_{t-1}^{ave} \left(1 - \frac{L_t}{D_t}\right)$$

$$F_t = (R_{t-1}^{ave} + \delta^d)D_{t-1}$$

Theoretical Framework Details 2

- Parsimonious: 2 state variables (D_t and R_t^{ave})
- Good fit on actual public debt promises.
- Rescale debt quantity variables in percent of GDP terms, with lower case letters being the value in GDP units $x_t \equiv \frac{X_t}{P_t Y_t}$ where P_t is the aggregate price level for consumption goods, Y_t is real GDP.
- Inflation is $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and real GDP growth is $g_t \equiv \frac{Y_t}{Y_{t-1}}$. [Back](#)

$$f_t = (R_{t-1}^{ave} + \delta^d) \frac{1}{g_t \pi_t} d_{t-1}$$

$$d_t = (1 - \delta^d) \frac{1}{g_t \pi_t} d_{t-1} + l_t$$

$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) R_{t-1}^{ave} + \frac{l_t}{d_t} R_t^{new}$$

Theoretical Framework Details 3

- Derive duration from δ^d and R_t^{new} .

$$Duration = \frac{1 + R_t^{new}}{\delta^d + R_t^{new}}$$

- Take s_t as the primary surplus in GDP units.
- If we have also non bond debt, s_t is net resource needs the government uses to cover bond payments.
- Close government system with budget constraint.

$$f_t = s_t + l_t$$

- Risk neutral investors price government bonds.

$$\frac{1}{(\delta^d + R_t^{new})} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\frac{1}{\pi_{t+j|t}} \beta^j (1 - \delta^d)^{j-1} \right]$$

- Market value of public debt: $q_t = \frac{\delta^d + R_t^{ave}}{\delta^d + R_t^{new}}$.

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Theoretical Framework Details 4

- Partial equilibrium approach. [Back](#)
- Exogenous processes for inflation, for inflation expectations, for primary surplus, and for growth.
- Given initial conditions for the state variables d_{-1} and R_{-1}^{ave} ,
- 6 endogenous variables $\{R_t^{new}, R_t^{ave}, l_t, f_t, d_t, q_t\}$ and corresponding equations:

$$f_t = (R_{t-1}^{ave} + \delta^d) \frac{1}{g_t \pi_t} d_{t-1} \quad \Leftarrow \text{Debt Payments}$$

$$d_t = (1 - \delta^d) \frac{1}{g_t \pi_t} d_{t-1} + l_t \quad \Leftarrow \text{Debt Law of Motion (LoM)}$$

$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) R_{t-1}^{ave} + \frac{l_t}{d_t} R_t^{new} \quad \Leftarrow \text{Average Interest Rate on Debt LoM}$$

$$f_t = s_t + l_t \quad \Leftarrow \text{Govt Budget Constraint}$$

$$\frac{1}{(\delta^d + R_t^{new})} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\frac{1}{\pi_{t+j|t}} \beta^j (1 - \delta^d)^{j-1} \right] \quad \Leftarrow \text{Bond Euler}$$

$$q_t = \frac{\delta^d + R_t^{ave}}{\delta^d + R_t^{new}} \quad \Leftarrow \text{Debt Market Value}$$

Theory Experiment Set-up

- Fiscal burden is the net present value of real primary surpluses $Y_t s_t$ evaluated with risk neutral utility

$$\sum_{j=1}^{\infty} \beta^j Y_{t+j} s_{t+j} \propto \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}$$

- Under rational expectations pricing, the expectation of the fiscal burden is the market value of debt.
- Define *parallel shifts or parallel derivatives* in future variables:

$$\frac{\partial y}{\{\partial x_{t+j}\}_{j=1}^m} \equiv \sum_{j=1}^m \frac{\partial y}{\partial x_{t+j}}$$

Theory Experiment Result 1: Parallel shifts versus blips in expectations: effect on new issuances

- If all inflation expectations long and short move (parallel shifts), then interest rates on newly issued debt will increase one to one.
- If only short run expectations move (blips), interest rates will increase by one over duration:

$$\frac{\partial R_{t+l}^{new}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} = \frac{1+R}{\pi}$$

$$\frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} = \frac{1+R}{\pi} \frac{1}{\frac{1+R}{\delta^d+R}}$$

Theory Experiment Result 2: Effect of changes in interest rate on the fiscal burden

- Impact of a parallel change in interest rates on newly issued debt on the average interest rates when the government only refinances the debt due.

$$\frac{\partial R_{t+k}^{ave}}{\{\partial R_{t+l}^{new}\}_{l=0}^k} = 1 - (1 - \delta^d)^{k+1}$$

- Effect on fiscal burden:

$$\begin{aligned} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\{\partial R_{t+l}^{new}\}_{l=0}^{\infty}} &= \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} \frac{\partial s_{t+j}}{\partial R_{t+j-1}^{ave}} \frac{\partial R_{t+j-1}^{ave}}{\{\partial R_{t+l}^{new}\}_{l=0}^k} \\ &= D \left(\frac{1}{R} - \frac{1+R}{\delta^d + R} \frac{1 - \delta^d}{1+R} \right) \end{aligned}$$

- D scales the change by the steady state values of the fiscal burden.
- The first term in parentheses is the permanent effect of a one percent increase in rates, the last term shows how much lower this will be if debt is longer maturity.
- First fraction of the second term is duration. The second term is a scaling factor that arises as scenario is an announced shock that will happen in the next period.

Theory Experiment Result 3: Effect of changes in inflation expectations (blips and parallel shifts) on the fiscal burden

$$\begin{aligned}
 \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} \\
 &= \frac{D}{\pi} \frac{1+R}{R} \delta^d \\
 \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} \\
 &= \frac{D}{\pi} \frac{1+R}{R} \delta^d \frac{1+R}{\delta^d + R}
 \end{aligned}$$

- If only short run expectations move, the effect on the debt burden of their increase is close to zero with long debt (δ^d small); not only legacy debt is insured, but also new debt does not change in cost.
- If all expectations move, then only legacy debt is insured against the hike and new debt will slowly embed the new expectations.

Theory Experiment Result 4: Effect of changes in inflation on the fiscal burden

- Turn to the effect of inflation, holding inflation expectations constant.
- Nominal rate on new debt is fixed as what matters are inflation expectations.
- As the government refinances the maturing debt, then also average interest rates do not depend directly on inflation.
- As interest rates are not affected by inflation, maturity also does not play a role.
- Effect on the fiscal burden is:

$$\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} = -\frac{D}{\pi} \frac{1+R}{R}$$

- Effect of inflation on the fiscal burden is larger the larger the debt stock, and as it is permanent, it will affect the whole future stream of surpluses. [Back](#)

Theory Experiment Result 5: Effect of changes in inflation and inflation expectations on the fiscal burden, blips and parallel shifts

$$\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} \equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} = -\frac{D}{\pi} \frac{1+R}{\delta^d + R} (1 - \delta^d) \frac{R + \delta^d}{R}$$

$$\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}} \equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^j g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})} = -\frac{D}{\pi} \frac{1+R}{\delta^d + R} (1 - \delta^d)$$

- When debt is short ($\delta^d = 1$), the effect is always zero.
- Example in quarterly model: interest rates at 5%, $R = 0.05/4$, duration at 6.75 years δ^d is 0.025, debt at 100% of GDP $D = 1 * 4$, inflation at 2% $\pi = 1.02^{1/4}$.
- $(1 - \delta^d)/\pi$ is quite close to one.
- DurGDP is 6.75% of GDP and $\frac{R+\delta^d}{R} = 3$ is large.
- Same pattern arises under an alternative experiment, where the government keeps a constant debt-to-GDP ratio by adjusting primary surpluses.

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Theory Experiment: Central Bank Fixes Interest Rates

- Not allowing any inflation expectation (at short or long horizons) to adjust after the change in inflation implies that nominal interest rates are fixed at all horizons.
- Use to study the implications of the central bank fixing interest rates, starting from steady state.
- Effect on fiscal burden is $-\frac{D}{\pi} \frac{1+R}{R}$ irrespective of maturity.
- Differs from baseline (fix long expectations, adjust short one) by $1 - \delta^d$. Small with plausible maturity calibration.
- Not much debt issued today is due in the next quarter. [Back](#)

	Central bank fixes rates $\{\partial \pi_{t+1+l}\}_{l=0}^{\infty}$	Long run expectations do not adjust $\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}$	Long run expectations adjust $\{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}$
Any Maturity δ^d	$-\frac{D}{\pi} \frac{1+R}{R}$	$-\frac{D}{\pi} \frac{1+R}{\delta^d + R} (1 - \delta^d) \frac{R + \delta^d}{R}$	$-\frac{D}{\pi} \frac{1+R}{\delta^d + R} (1 - \delta^d)$
Short Debt $\delta^d = 1$	$-\frac{D}{\pi} \frac{1+R}{R}$	0	0

Computational Exercise Details - Data 1

- Same dataset as in the empirical exercises.
- For bond debt-to-GDP d_t , average interest rates at book value R_t^{ave} , and average fraction of bond debt due in each period δ_t^{ave} , we use seasonally adjusted government securities debt over GDP (OECD), government bonds average coupon (WGBI), and (one over) government bonds average life (WGBI).
- Real GDP growth g_t (OECD data for European Countries and Fred for the US) and inflation π_t (HICP for the Euro Area and CPI for the US) are QoQ rates.
- For inflation expectations, we use SPF data and we assign forecasts in the following way.
 - For the Euro Area expectations, we use current year expectations for the following two quarters, one year ahead expectation for three and four quarters ahead, two years ahead expectations for five to eight quarters ahead, and long run expectations from nine quarters ahead onwards.
 - For the US, we use the same data, but as two years ahead expectations are not recorded in the SPF we use inflation in two calendar years from now for 5 to 8 quarters ahead when available and QoQ inflation 4 quarters ahead when not available.

Computational Exercise Details - Data 2

- For interest rate on newly issued debt R_t^{new} , we use the benchmark interest rate on a 10 years government bond.
- We compute issuance over GDP I_t and (one over) maturity on newly issued debt δ_t^{new} from model equations.
 - This is “flow” derived data from “stock” original data, so it is imprecise. Therefore, we filter δ_t^{new} with a HP filter with a low smoothing parameter (10) and keep the trend component.
 - We ensure that the maximum average maturity for newly issued debt is 20 years (we allow for longer debt, up to consols, but the average maturity of debt issued in a quarter does not exceed that threshold, as in the data).
- We calibrate β as 0.995.
- Compute convenience yield (or term, liquidity, or risk premium) as the difference between interest rates on newly issued debt and the interest rate that would prevail under risk neutral pricing from the Euler equation.
- Set the net resource needs s_t from budget constraint and model equations.

Computational Exercise Details - Model

- Exogenous processes from data for inflation, for inflation expectations, for primary surplus, growth, for convenience yield c_t , and for maturity of new debt δ_t^{new} . [Back](#)
- Given initial conditions for the state variables d_{-1} , R_{-1}^{ave} , and δ_{-1}^{ave} ,
- 6 endogenous variables $\{R_t^{new}, R_t^{ave}, l_t, f_t, d_t, \delta_{t-1}^{ave}\}$ and corresponding equations:

$$f_t = (R_{t-1}^{ave} + \delta_{t-1}^{ave}) \frac{1}{g_t \pi_t} d_{t-1} \quad \Leftarrow \text{Debt Payments}$$

$$d_t = (1 - \delta_{t-1}^{ave}) \frac{1}{g_t \pi_t} d_{t-1} + l_t \quad \Leftarrow \text{Debt Law of Motion (LoM)}$$

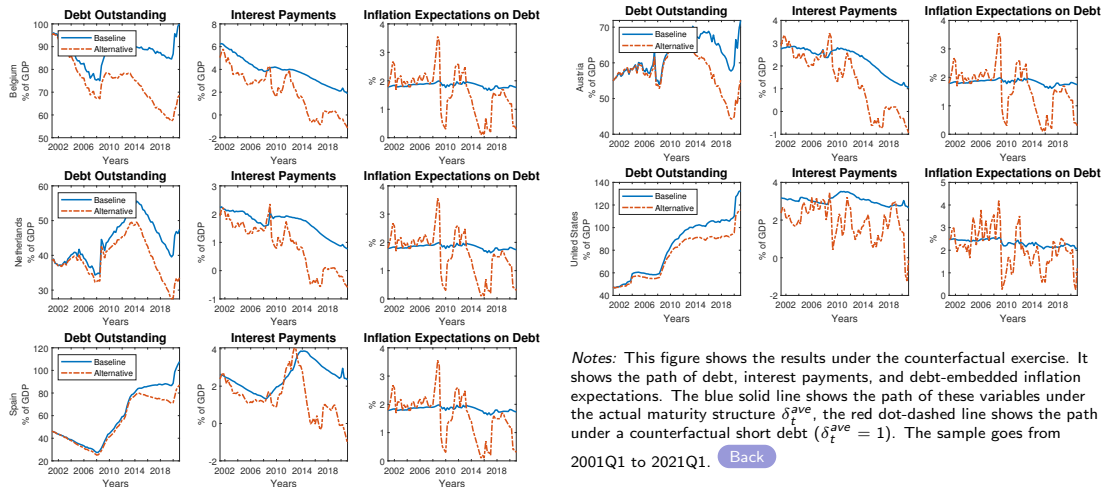
$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) R_{t-1}^{ave} + \frac{l_t}{d_t} R_t^{new} \quad \Leftarrow \text{Average Interest Rate on Debt LoM}$$

$$\delta_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) \delta_{t-1}^{ave} + \frac{l_t}{d_t} \delta_t^{new} \quad \Leftarrow \text{Average Debt Maturity LoM}$$

$$f_t = s_t + l_t \quad \Leftarrow \text{Govt Budget Constraint}$$

$$\frac{1}{(\delta_t^{new} + R_t^{new} - c_t)} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\frac{1}{\pi_{t+j|t}} \beta^j (1 - \delta_t^{new})^{j-1} \right] \quad \Leftarrow \text{Bond Euler}$$

Computational Exercises



Notes: This figure shows the results under the counterfactual exercise. It shows the path of debt, interest payments, and debt-embedded inflation expectations. The blue solid line shows the path of these variables under the actual maturity structure δ_t^{ave} , the red dot-dashed line shows the path under a counterfactual short debt ($\delta_t^{ave} = 1$). The sample goes from

2001Q1 to 2021Q1.

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Computational Exercises - Longer Sample

Country	Short Debt		Perfect Foresight	
	Debt	Interest Payments	Debt	Interest Payments
France	20.6	1.21	2.2	0.14
Germany	13.2	0.78	1.7	0.11
Italy	30.2	1.69	3.3	0.19
Belgium	29.7	1.86	2.5	0.16
Spain	20.0	1.13	1.9	0.11
Netherlands	13.0	0.80	1.3	0.08
Austria	15.4	0.90	2.0	0.13
United States	14.4	1.05	2.6	0.20

Sample: 2001Q1 - 2022Q1

Notes: This table shows the results under the counterfactual exercises. Columns 2 and 3 show the counterfactual fiscal burden under a short debt profile ($\delta^d = 1$). Columns 4 and 5 show the counterfactual fiscal burden under perfect foresight, that is in each period t expectations are correct at all future horizon: $\mathbb{E}_t(\pi_{t+j}) = \pi_{t+j}$. For inflation expectations pertaining to periods that have not yet happened in the dataset (after 2022Q1) we use the appropriate inflation expectation. Columns 2 and 4 show the difference in debt-to-GDP level at the last period under the exercise compared to the case where we fit the model with actual data. Columns 3 and 5 show the difference in average interest payments per year under the exercise compared to the case where we fit the model with actual data. The sample goes from 2001Q1 to 2022Q1.

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Counterfactual Exercises - Alternative Inflation Measures

Country	Baseline		CS HICP		GDP Deflator	
	Debt	Interest Payments	Debt	Interest Payments	Debt	Interest Payments
France	21.4	1.20	21.6	1.21	21.5	1.21
Germany	13.9	0.78	13.7	0.78	13.3	0.78
Italy	31.2	1.63	31.9	1.64	30.5	1.60
Belgium	31.2	1.86	30.3	1.85	29.1	1.80
Spain	20.6	1.09	20.8	1.09	19.5	1.03
Netherlands	13.9	0.81	13.6	0.80	13.6	0.81
Austria	16.7	0.90	15.0	0.84	16.1	0.88
United States	17.0	1.14			17.2	1.14

Notes: This table shows the results under the counterfactual exercises under different measures for realised inflation. All show the counterfactual fiscal burden under a short debt profile ($\delta^d = 1$) compared to the actual maturity. Columns 2 and 3 show the baseline metric for realised inflation Euro Area wide HICP for the Euro Area countries and CPI for the US. Columns 4 and 5 show use Country Specific (CS) HICP for Euro Area countries realised inflation. Columns 6 and 7 show use country specific GDP deflator for Euro Area countries and for the United States for realised inflation. Columns 2, 4, and 6 show the difference in debt-to-GDP level at the last period. Columns 3, 5, and 7 show the difference in average interest payments per year. The sample goes from 2001Q1 to 2022Q1.

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Computational Exercises Comparison

- Compare numbers in the short-vs-long debt (1) scenario with those in perfect inflation foresight-vs-actual inflation expectations (2):
 - In (1), debt burden falls faster because the secular decline in interest rates is immediately incorporated in average interest rates, whereas with long debt, even with perfect foresight, the decline is incorporated only on newly issued debt and not on legacy debt.
 - (2) captures “mistakes” in expectations, but does not adjust for the fact that realised lower inflation increases the real debt burden on legacy debt. (1) does.
 - In (1), we used interest rates on 10 year bonds for R_t^{new} (*ceteris paribus* comparison), with short (3 months) rates, the differential in short-vs-long debt would have been even higher as the yield curve was generally upward sloping (term premium).
- Results are context specific. Methodology is general. [Back](#)