Online Appendix
(not intended for publication)

"Non-Essential Business Cycles"

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F Measurement

This Appendix first outlines the approach for constructing new data series for essentials and non-essentials. As a first step, we classify goods and services into non-essentials and essentials, using CEX data. With this classification, we construct novel consumption and price series using PCE data. Next, using our final goods and services split across essentials and non-essentials and the input output tables, we classify industries according to the final goods and services that they ultimately sell to downstream. This industry classification is used to construct labour market series with CPS data. We also present additional macroeconomic data sources, summary statistics, alternative consumption categorisations, and the state-level analysis.

F.1 Classification procedure for consumption categories

This section outlines how we classify consumption categories into non-essentials and essentials, to build time series for consumption, prices, and labour market variables. Our first step uses consumption micro-data from the Consumer Expenditure Survey (CEX) to define what types of consumption goods are non-essential vs essential.

We classify consumption categories into into essentials and non-essentials by estimating Engel Curves closely following the approach used by Aguiar and Bils (2015). Aguiar and Bils (2015) use household microdata from three waves of the CEX and estimate the expenditure elasticities as β_j from:

$$\ln x_{hjt} - \ln \bar{x}_{hjt} = \alpha_{jt} + \beta_j \ln X_{ht} + \Gamma_j \mathbf{Z}_h + u_{hjt} \tag{4}$$

Where x_{hjt} is the expenditure by household h on goods of type j in year t, $\bar{x}_{hjt}$ is the equivalent average across households, X_{ht} is total household expenditure, instrumented by household income (dummies for category and log real after-tax income), α_{jt} are good fixed effects and $\mathbf{Z}_h$ are household characteristics (age range, earners and household size). For full details see the original paper, we replicate the identical empirical specification.

In Table A.1, we report the estimated expenditure elasticities and expenditure shares for the revised goods categories, which is a replication of Table II of Aguiar and Bils (2015), omitting the final two columns[1]. Essentials are defined as categories with an income/total expenditure elasticity of demand (β_j) less than one; non-essentials are defined as those with an elasticity greater than one.

We make two minor alterations to Aguiar and Bils (2015)'s approach. Firstly, we alter slightly the set of product categories, introducing some narrower categories where the broader categories included goods that varied considerably in their elasticities. Specifically,

[1] Aguiar and Bils (2015) use two specifications, using either income to instrument total expenditure, or lagged total expenditure to instrument current total expenditure. We use the former here. The reason is to hedge against the possible concern that the lagged spending instrument might bias downward the estimated elasticity for lumpy expenditure sectors, such a new cars. Whenever a household buys a car, they have higher total expenditure in that quarter, but the predicted expenditure from the instrument of the last quarter is lower, therefore associating a higher car expenditure with a lower total predicted expenditure, which biases the IED estimate towards a necessity. This attenuation in the elasticity estimate is not present with the income instrument, and in practice makes a substantial difference to the estimated IED for new cars.

we split "Applicances, phones, computers with associated services" into "Communications" and "Household appliances", "All other transportation" into "Gas and vehicle maintenance" and "Public transport", "Housing" into "Rents" and "Owner-occupied housing consumption" and "Vehicle purchasing, leasing and insurace" into "New car purchases", "Used car purchases" and "Other car spending (leasing, financing and insurance". We also omit tobacco from the product categories, as the intertemporal substitutability of tobacco is likely more related to the addictive nature of the good than the income elasticity, so less related to our theoretical framework. Secondly, we estimate the Engel curves for 1995-1997 rather than 1994-1996, in order to use the more consistent goods categories reported in the CEX Interview FMLI files during these years. As Aguiar and Bils (2015) note, the expenditure elasticities do not vary considerably over time, and consistent with this using the slightly different sample period makes minimal difference to their original estimated elasticities.

Table F.1 shows the expenditure shares of non-essentials vs essentials by housing tenure type and by income group using the elasticities above, on the same CEX sample. For mortgagors the non-essential share is 63.9%, for owner-occupiers without a mortgage this is 60.6% and for renters it is 33.6%. Households in the lowest income tercile have a non-essential share of 44.3% and households in the top two income terciles have a non-essential share of 60.3%. We use this information to calibrate the structural model, as detailed in Table 1. Note that the expenditure shares here differ slightly from consumption shares reported from PCE data. There are two main reasons for this; i) expenditure shares here reflect nominal expenditure shares, rather than real consumption shares constructed from chained consumption series and ii) because of the differences in the underlying data.

Table F.1: Non-essential expenditure shares: by tenure type and across income distribution

	Non-essential share	
By housing tenure type		
Mortgagor	63.9%	
Owner occupier (without mortgage)	60.6%	
Renter	33.6%	
	Non-essential share	
By income tercile		
First	44.3%	
Second	56.1%	Top 2/3: 60.3%
Third	63.3%	
	Non-essential share	
By income quintile		
First	43.1%	
Second	48.7%	
Third	55.8%	
Fourth	59.8%	
Fifth	64.9%	

Notes: Non-essential expenditure shares from CEX data (see text). Income terciles and quintiles are based on after tax income.

F.2 Construction of Consumption and Price Indices

In this subsection, we show how we construct time series for consumption and price indices.

Using the estimated elasticities and classification into essential and non-essentials from the previous section, we match their counterparts in the *PCE by Type of Product* tables from the U.S. Bureau of Economic Analysis (BEA). The consumption categories included in the above do not cover the entire consumption bundle of households, but our approach is to maximise the coverage as much as possible. This mapping closely follows a similar exercise in Aguiar and Bils (2015). These omissions and adjustments largely follow Aguiar and Bils (2015) and include cases where either:

1. Expenditures not made entirely by private, US households for their own personal consumption; if they are made on behalf of households by non-profits, employers or insurers.
   - Includes: food on farms, food supplied to military, net expenditures abroad, expenditures relating to net foreign travel, final consumption expenditures of nonprofit institutions serving households, some categories of insurance.
2. The expenditure might reasonably not be considered consumption which generates personal utility, and is instead a form of saving or cost of saving or other expense.
   - Includes: financial services (bank/pension fund fees, investment service commissions), some categories of insurance.
3. We don't trust or unable to estimate reasonable Engel curve estimates using the CEX microdata, due to incomplete or inaccurate consumption reporting.
   - Includes: professional and other services (legal, accounting, union, professional associations, funerals), Foundations and grantmaking and giving services to households.
4. We classify children's clothing as essential and adults clothing as non-essential, using CEX data. In the PCE, there are three clothing categories; 'Women's and girls' clothing', 'Men and boys; clothing', and 'Children's and inflant's clothing'. We follow Aguiar and Bils (2015) in splitting the former two categories, attributing 22% to children's, essential clothing, and 78% to adults, non-essential clothing.
5. For health expenditures, we also follow Aguiar and Bils (2015) in only including the proportion of health expenditure made out of pocket by households, by adjusting down the health expenditure and net health insurance expenditures using National Health Expenditure Data from Centers for Medicare and Medicaid Services. This helps reduce the proportion of health expenditure which is contributed to by (for instance) government programmes and so not discretionary spending by households directly, but still included in PCE.

Following this process, we classify on average over the sample period 36% of expenditure reported in the PCE as essential, 44% as non-essential and the remaining 20% is left unclassified.

We then construct Fisher price and consumption quantity indices for essentials and non-essentials by aggregating the (nominal) expenditure and price subindices following the approach outlined NIPA (2021), Chapter 4. The quantity index aggregated from all the subindices i categorised as essentials (E) is given by:

$$Q^F_{t,E} = \sqrt{\frac{\sum_{i \in E} p_{i,t-1} q_{i,t}}{\sum_{i \in E} p_{i,t-1} q_{i,t-1}} \times \frac{\sum_{i \in E} p_{i,t} q_{i,t}}{\sum_{i \in E} p_{i,t} q_{i,t-1}}}$$

Where the (deflated) values within the summations are calculated using the nominal expenditure $e_{i,t}$ and price indices $p_{i,t}$ as appropriate, for instance:

$$p_{i,t-1} q_{i,t} = p_{i,t-1} * \frac{p_{i,t} q_{i,t}}{p_{i,t}} = p_{i,t-1} * \frac{e_{i,t}}{p_{i,t}}$$

And similarly for different combinations of lagged quantities and prices.

We construct the Fisher price indices for essentials as:

$$P^F_{t,E} = \sqrt{\frac{\sum_{i \in E} p_{i,t} q_{i,t-1}}{\sum_{i \in E} p_{i,t-1} q_{i,t-1}} \times \frac{\sum_{i \in E} p_{i,t} q_{i,t}}{\sum_{i \in E} p_{i,t-1} q_{i,t}}}$$

And the equivalent formulas for non-essentials. When we refer to consumption shares with the PCE data, we use chained consumption series also following the NIPA guidelines.

F.3 Mapping of final goods classification to industries for labour market variables

The next step in the process is to use the classification of the consumption goods to understand which industries are producing non-essentials vs essentials. This industry classification will allow us to classify workers into the sectors they work for and understand the labour market implications of non-essential consumption dynamics. The first step to do this is to classify final goods producing industries according to the goods and services they supply. However, we would also like to classify intermediate industries, in order to also account for upstream labour market implications of final good demand. To achieve this second step, we use the input-output matrix from the BEA to understand the downstream final goods that intermediate industries contribute to. The final step we take is to use this classification with CPS data to build time series of labour earnings, employment, and wages of worker who mainly produce essentials and non-essential goods and services.

Final goods producer classification. The first stage of this process is the final goods classification. We map consumption categories to all NAICS 2007 industries included in the input-output tables of the BEA. We manually classify all industry codes as either essential, non-essential or unclassified, based on whether the industry produces final consumption goods which fit into our classified consumption categories.

We take an unconservative approach to this final goods industry classification, in order to maximise the amount of employment we are able to categorise. If there is an industry which is primarily producing intermediate goods, but related to one consumption category, we still classify it according to that consumption category. This is because our second step using the input output approach we use will reassign an industry's sales of input goods to different sectors according to their eventual downstream use. For example, 'Photographic and Photocopying Equipment Manufacturing' (NAICS code 333316) would be an non-essential if purchased by households, but supplies a lot of intermediate inputs which are used in essential industries, so this is eventually classified as a essential industry. Sometimes we classify industries that produce a range of goods to the consumption category which they are *most* rather than entirely associated with. For instance, employees working for department stores may supply both essential and non-essential consumption goods, but we assume that the majority of goods supplied are within the non-essential consumption categories, and so classify these as non-essential.

Input-Output approach to classify intermediate industries. We would like to be classify industries which primarily produce intermediate goods based on the downstream final goods that they primarily supply. In order to do this, we use the input-output tables combined with the final good industries from the previous section. We take the *Use of commodities by industry* table from the BEA Input-Output Accounts Data for 2007 at the most detailed disaggregation of 405 industries. From there we exclude government, private households, secondary smelting and alloying of aluminum, scrap, used and secondhand goods, noncomparable imports, and rest of the world adjustment. This allows to have a square matrix of input-output linkages with 391 industries both as suppliers and buyers of intermediate inputs. We link each intermediate industry to the final products with the Leontief inverse, in order to assign each industry the essential or non-essential final products. For categories that we do not have downstream sales data, we use the final product classification from the CEX.

A simple production network model in the spirit of Acemoglu et al. (2012) can help to explain all the steps. We take an economy with N industries comprising intermediate and final products. Each industry i has total sales $X_i = p_i x_i$ which can be made to intermediate producers $p_i x_{i,j}$, consumers for personal consumption expenditures $C_i = p_i c_i$ or other agents for final good expenditures $Z_i = p_i z_i$ (these can be government, investment, inventories, or exports). Total quantity sold is:

$$x_i = \sum_{j=1}^{N} x_{i,j} + c_i + z_i$$

The production function of industry j uses intermediate inputs $x_{i,j}$ and other inputs l_j in order to produce x_j with a Cobb-Douglas production function:

$$x_j = A_j l_j^{\alpha_j} \prod_{i=1}^{N} x_{i,j}^{(1-\alpha_j)\omega_{i,j}}$$

The first order condition under perfect competition for each intermediate input is: $p_i x_{i,j} =$

$p_j x_j (1-\alpha_j)\omega_{i,j}$. This allows a recursive structure on the industry sales by substituting it in:

$$X_i = \sum_{j=1}^{N} (1-\alpha_j)\omega_{i,j} X_j + C_i + Z_i$$

Which we can write in matrix form and invert it to find the Leontief inverse L. Notice that we use $\circ$ for the Hadamard product (the element-wise product).

$$\begin{aligned} X &= (((1-\alpha)\mathbf{1}'_N) \circ \Omega)X + C + Z \\ X &= (I_N - ((1-\alpha)\mathbf{1}'_N) \circ \Omega)^{-1}(C+Z) \\ X &= L(C+Z) \end{aligned}$$

We have a classification of final products as essential E, non-essential N, or unclassified U we can build three $N \times 1$ indicator vectors taking value one if the final product is of that category and zero otherwise: $\mathbb{1}_k$ for $k = \{E, N, U\}$. We can assign an industry to essential if this industry sells more to essential final goods than non-essential final goods and if the sum of these sales is higher than the sales to unclassified sectors. Mathematically, we assign industry i to essentials if:

$$\begin{aligned} \{L(C \circ \mathbb{1}_E)\}_i &> \{L(C \circ \mathbb{1}_N)\}_i \\ \{L(C \circ \mathbb{1}_E)\}_i + \{L(C \circ \mathbb{1}_N)\}_i &> \{L(C \circ \mathbb{1}_U)\}_i \end{aligned}$$

And similarly for non-essentials. We leave as unclassified each remaining industry. Intuitively this method allows to match intermediate industries to their most important final goods. As an example, we match *Grain farming* to essentials, and *Iron, gold, silver, and other metal ore mining* to non-essential, despite not being classified within final goods (as they are intermediates).

Given the intermediate input-output matrix cleaned with the steps above, $((1-\alpha)\mathbf{1}'_N) \circ \Omega$ is the IO matrix with each intermediate input sales $p_i x_{i,j}$ divided by the *Total industry output (basic value)* line: $p_j x_j$. The C vector we use to weight each sales to assign to the three categories is *Personal consumption expenditures* in the input-output data.

The outcome of this exercise is the classification in essentials and non-essentials of the intermediate and final industries, defined with NAICS 2007 codes.

Mapping between industry codes. Our objective is to create time series for labour market variables split by essentials and non-essentials, e.g. what are the labour earnings of workers who predominantly produce non-essentials. However, we must overcome one last intermediate step before merging the industry classification with the labour market data from the CPS: the datasets we use to classify industries and workers use different industry codes. To accommodate this, we have to map between two different industry codes; NAICS 2007 and census 1990. Table F.2 shows the steps we follow.

We primarily use the the cross-walk supplied by the Census Bureau for this. However, sometimes we use some discretion and make some assumptions to do map between the codes. First, we classify NAICS 2007 codes to categorise industries according to our essential/non-

Table F.2: Datasets and industry codes for labour market classification

CEX Table A.1	Classify industries →	Input-Output NAICS 2007	Adjust industry codes →	CPS Census 1990

Notes: This table shows the different dataset we use and the corresponding industry codes classification to classify labour market variables. Arrows show the direction of the mapping, from the initial final good classification to the final time series.

essential split from the CEX. Then we use these NAICs codes in our input-output adjustment process to classify intermediate industries. Once armed with intermediate industry classifications using the input-output approach, we then map the classification to census 1990 codes. This mapping between industry codes requires some approximations and adjustments:

1. Most importantly, for retail industry codes (census codes 580-691), many of the census codes are more disaggregated than the available NAICS codes. For those, we overwrite the intermediate industry classification from the input-output process, and instead we use the initial classification of the industry. This is because these industries primarily supply final goods which are more straightforward to classify directly than intermediate industries. We also directly classify private households as non-essential, as this is also a exclusively final goods industry.

2. A portion of NAICS codes have multiple NAICS codes in the industry data for one census code. An example of this is dairy product manufacturing (census code 101) which in the input output tables maps to four NAICS industry categories (Cheese manufacturing; Dry, condensed, and evaporated dairy product manufacturing; Fluid milk and butter manufacturing; Ice cream and frozen dessert manufacturing). For these cases, we apply the same classification for all NAIC codes that related to a particular census code, treat them as separate industries in the input-output table processing, and then average the final sales shares to different categories of industries (essential, non-essential and unclassified) across a census industry using the total sales of each NAICS industry as weights.

3. Some census codes are more detailed than the NAICS codes in the input-output tables. For example, there is a census code (402) for taxicab services, which corresponds to NAICS code 485300 but only the more aggregated NAICS code 485000 is available in the input-output tables. In these cases, we assign the sales shares of the more aggregated NAICS industry to the more disaggregated census industry. This assumes that the disaggregated industry does not vary substantially in what it supplies goods to compared to the more aggregated industry.

4. Some census codes are only mapped to large NAICS categories in the crosswalk, often because they are non-specificed or miscellaneous industries. For example, the census code 472 (non-specified utilities) is part of NAICS code 22, although there are more direct mappings between the codes in NAICS 22 and the census codes. For those industries, we also take an weighted average of all sales shares of all relevant industries (here, for example, 221100, 221200 and 221300), again assuming that the average of

the larger group will be representative of the industries in the census code. Where not possible, (in particular, for Manufacturing non-durable, allocated) we leave unclassified.

5. Finally, there a few remaining cases where the mapping is less straightforward, because industries are divided differently in the two industry classifications. For example, knitting mills (census code 132) corresponds to NAICS codes 31324 and 3151, but in the input-output tables only the larger categories 3132 and 315 are available. In the same spirit as the previous approaches, we select all NAICS codes at the more aggregated level that include relevant industries, and take a total sales-weighted average of the sales shares to essentials, non-essentials and apply this to the census industry. Again this assumes that the census industry's sales shares are represented reasonably by the more aggregated industry.

Full mappings between NAICS 2007 industries in the input-output tables and the 1990 census industry codes used are given in the replication files.

Final classification of industries into essentials and non-essentials. Using the classification from the Input-Output approach we classify all industries as either essential, non-essential or unclassified. The final industry classification is presented in Table F.3. This is the classification we use for labour market variables.

Table F.3: Industry classification

Essential
Coal mining; oil and gas extraction; meat products; dairy products; canned, frozen, and preserved fruits and vegetables; grain mill products; bakery products; sugar and confectionery products; misc. food preparations and kindred products; food industries, n.s; miscellaneous paper and pulp products; drugs; soaps and cosmetics; agricultural chemicals; industrial and miscellaneous chemicals; petroleum refining; miscellaneous petroleum and coal products; tires and inner tubes; farm machinery and equipment; construction and material handling machines; office and accounting machines; guided missiles, space vehicles, and parts; medical, dental, and optical instruments and supplies; photographic equipment and supplies; u.s. postal service; pipe lines, except natural gas; wired communications; telegraph and miscellaneous communications services; electric light and power; gas and steam supply systems; electric and gas, and other combinations; water supply and irrigation; sanitary services; utilities, n.s; professional and commercial equipment and supplies; drugs, chemicals, and allied products; groceries and related products; petroleum products; wholesale trade, n.s; grocery stores; dairy products stores; food stores, n.e.c; auto and home supply stores; gasoline service stations; drug stores; fuel dealers; retail florists; insurance; personnel supply services; automobile parking and carwashes; automotive repair and related services; beauty shops; barber shops; funeral service and crematories; miscellaneous personal services; offices and clinics of physicians; offices and clinics of dentists; offices and clinics of chiropractors; offices and clinics of optometrists; offices and clinics of health practitioners, n.e.c; hospitals; nursing and personal care facilities; health services, n.e.c; residential care facilities, without nursing; accounting, auditing, and bookkeeping services; management and public relations services
Non-essential

Metal mining; nonmetallic mining and quarrying, except fuels; all construction; beverage industries; knitting mills; dyeing and finishing textiles, except wool and knit goods; carpets and rugs; yarn, thread, and fabric mills; miscellaneous textile mill products; apparel and accessories, except knit; miscellaneous fabricated textile products; pulp, paper, and paperboard mills; paperboard containers and boxes; newspaper publishing and printing; printing, publishing, and allied industries, except newspapers; plastics, synthetics, and resins; paints, varnishes, and related products; other rubber products, and plastics footwear and belting; miscellaneous plastics products; leather tanning and finishing; footwear, except rubber and plastic; leather products, except footwear; logging; sawmills, planing mills, and millwork; wood buildings and mobile homes; miscellaneous wood products; furniture and fixtures; glass and glass products; cement, concrete, gypsum, and plaster products; structural clay products; pottery and related products; misc. nonmetallic mineral and stone products; blast furnaces, steelworks, rolling and finishing mills; iron and steel foundries; primary aluminum industries; other primary metal industries; cutlery, handtools, and general hardware; fabricated structural metal products; screw machine products; metal forgings and stampings; ordnance; miscellaneous fabricated metal products; metal industries, n.s; engines and turbines; metalworking machinery; computers and related equipment; machinery, except electrical, n.e.c; machinery, n.s; household appliances; radio, tv, and communication equipment; electrical machinery, equipment, and supplies, n.e.c; electrical machinery, equipment, and supplies, n.s; motor vehicles and motor vehicle equipment; aircraft and parts; ship and boat building and repairing; railroad locomotives and equipment; cycles and miscellaneous transportation equipment; toys, amusement, and sporting goods; manufacturing industries, n.s; railroads; bus service and urban transit; taxicab service; warehousing and storage; water transportation; air transportation; services incidental to transportation; radio and television broadcasting and cable; motor vehicles and equipment; furniture and home furnishings; lumber and construction materials; metals and minerals, except petroleum; electrical goods; hardware, plumbing and heating supplies; machinery, equipment, and supplies; scrap and waste materials; miscellaneous wholesale, durable goods; paper and paper products; apparel, fabrics, and notions; farm-product raw materials; alcoholic beverages; farm supplies; miscellaneous wholesale, nondurable goods; lumber and building material retailing; hardware stores; retail nurseries and garden stores; mobile home dealers; department stores; variety stores; miscellaneous general merchandise stores; retail bakeries; motor vehicle dealers; miscellaneous vehicle dealers; apparel and accessory stores, except shoe; shoe stores; furniture and home furnishings stores; household appliance stores; radio, tv, and computer stores; music stores; eating and drinking places; liquor stores; sporting goods, bicycles, and hobby stores; book and stationery stores; jewelry stores; gift, novelty, and souvenir shops; sewing, needlework, and piece goods stores; catalog and mail order houses; vending machine operators; direct selling establishments; miscellaneous retail stores; retail trade, n.s; savings institutions, including credit unions; credit agencies, n.e.c; real estate, including real estate-insurance offices; advertising; services to dwellings and other buildings; computer and data processing services; detective and protective services; business services, n.e.c; automotive rental and leasing, without drivers; electrical repair shops; miscellaneous repair services; private households; hotels and motels; lodging places, except hotels and motels; laundry, cleaning, and garment services; shoe repair shops; dressmaking shops; theaters and motion pictures; bowling centers; miscellaneous entertainment and recreation services; elementary and secondary schools; colleges and universities; vocational schools; educational services, n.e.c; child day care services; family child care homes; museums, art galleries, and zoos; labor unions; religious organizations; membership organizations, n.e.c; engineering, architectural, and surveying services; miscellaneous professional and related services.
Unclassified
Tobacco manufactures; manufacturing, non-durable - allocated; scientific and controlling instruments; watches, clocks, and clockwork operated devices; miscellaneous manufacturing industries; trucking service; banking; security, commodity brokerage, and investment companies; legal services; libraries; job training and vocational rehabilitation services; social services, n.e.c; research, development, and testing services

Notes: Classification of 1990 Census industry codes into essential, non-essential and unclassified.

Classification of labour market variables. Our last step is to use microdata from the Current Population Survey (CPS) to construct labour market series across industries; e.g. employment in the sectors producing essential good and services or labour earnings in the sectors producing non-essential good and services.

We construct employment using the main sample, and weekly usual earnings from the CPS ORG sample. We omit all workers working in agriculture or for the government. We combine these two series to give overall labour earnings for each sector. We also calculate earnings distributions within each sector, based on weekly usual earnings, as described in the main text. Using this classification, over the sample period 62% of employment is classified as non-essential, 30% as essential and the remaining 8% is unclassified.

Rather than the binary classification of industries into essential and non-essential, an alternative approach would be to classify the *share* of an intermediate industry which supplies downstream to non-essentials. For instance if a worker is employed in an industry where 60% of downstream consumption is essential 30% is non-essential and the remainder unclassified, in our baseline series we classify this employee as one essential worker. In our shares series, the employee would be counted as 0.6 of a person in the essential total employment series and 0.3 of a person in the non-essential employment series. We verify that our baseline empirical results are robust to using this alternative approach (results available upon request).

F.4 Other macroeconomic data sources

In addition to the constructed non-essential and essential series for consumption, prices, employment and earnings, we also use additional aggregate macroeconomic time-series in our Proxy-SVAR and local projection estimation, the sources for which are detailed below.

In the Proxy SVAR:

- Industrial production (INDPRO), PCE price index (PCEPI) and end of month 1y Treasury yields (DGS1) - downloaded from St Louis Fed's FRED, specific variable names in brackets.
- Excess bond premium, from the Federal Reserve Board[2]
- Monetary policy surprise series - both taken from the replication files of Jarociński and Karadi (2020):
  - The Gertler and Karadi shocks we use are the FF4 surprises updated and provided by Jarocinski and Karadi, which go from 1990m2 to 2016m12. There is a missing value on 2001m9 which we fill as zero.
  - The Jarconski and Karadi shocks we use, mitigating the information effect, are the FF4 surprise if there is a negative correlation between the FF4 surprise and the SP500 surprise. These go from 1990m2 to 2016m12. There is a missing value on 2001m9 which we fill as zero.

In the smooth local projection estimations, in addition to the Proxy SVAR, we add:

[2]https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/updating-the-recession-risk-and-the-excess-bond-premium-20161006.html

- Total employment - depending on the sample, this is aggregated from the CPS data described previously for employment and earnings IRFs, otherwise we use total private employment recorded by the Current Employment Statistics (Establishment Survey, CES), taken from FRED (variable name USPRIV).

- Overall earnings - to compare with our constructed earnings series, we use the BEA NIPA series Total Compensation of Employees (Received: Wage and Salary Disbursements)

- Per worker earnings - median earnings series constructed using CPS data described previously, for SLP-IV IRFs for earnings. Otherwise, to give a longer time-series, we use Average Weekly Earnings of Production and Nonsupervisory Employees, for Private employees from the CES, also taken from FRED (CES0500000030).

- For the price IRFs, we also use inflation expectations as an additional control. For these, we use University of Michigan Inflation expectations, also taken from FRED (MICH)

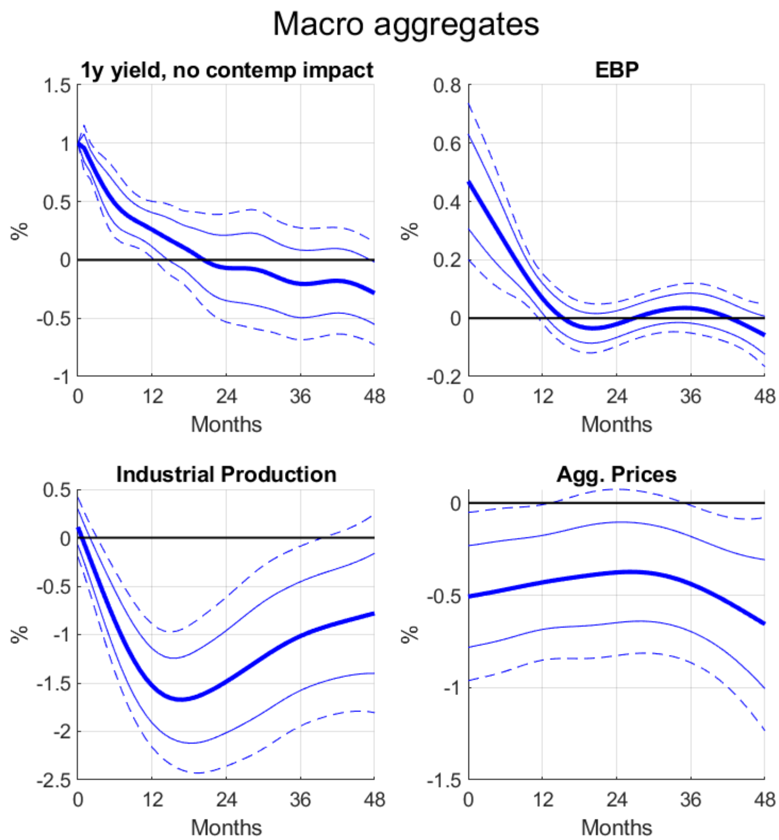
F.5 IRFs for other macro aggregates

Figure F.1 shows the IRFs estimated using our SLP-IV specification for the other macroeconomic aggregate series used as controls and in the Proxy-SVAR. These are 1y yields, the excess bond premium, industrial production and the PCE price index. The results are broadly consistent with standard responses, for instance those given in Gertler and Karadi (2015) using their HFI instrument and SVAR. The shock is a 100bp exogenous rise in 1y yields, after which 1y yields fall and here fall significantly below their prior level by four years after the shock, rather than reverting back to their prior level. The excess bond premium rises about half the amount of 1y yields, but reverts to zero by 18 months after the shock. Industrial production falls by 2% by 15 months after the shock before recoving and becoming insignificantly different from zero by three years after the shock. Aggregate prices fall insignificantly.

F.6 Quarterly earnings IRFs

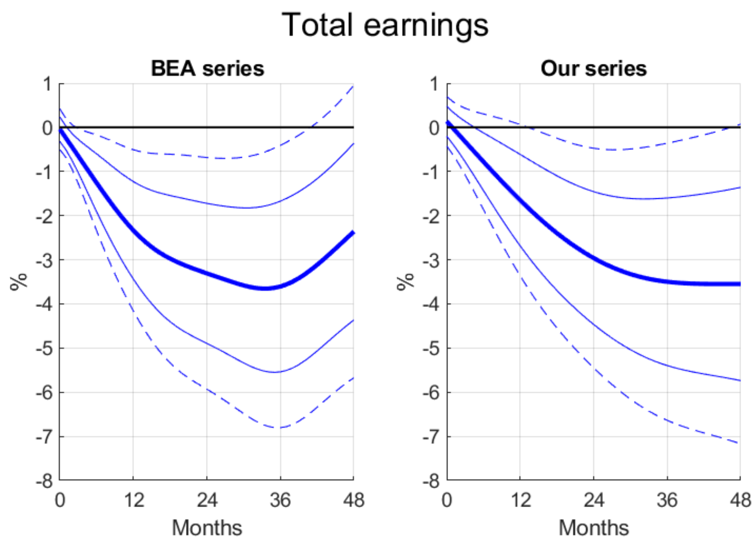
The CPS ORG sample is formally designed to be representative only at the quarterly frequency, but in our main results we use monthly frequency. To verify our results still hold at the lower frequency, Figure F.3 shows our main results for earnings using quarterly frequency data. As the quarterly frequency removes some of the higher frequency variation useful for identifying responses, the results are less significant but qualitatively similar to the baseline results.

Figure F.1: IRFs to contractionary monetary policy shock - Macro aggregates



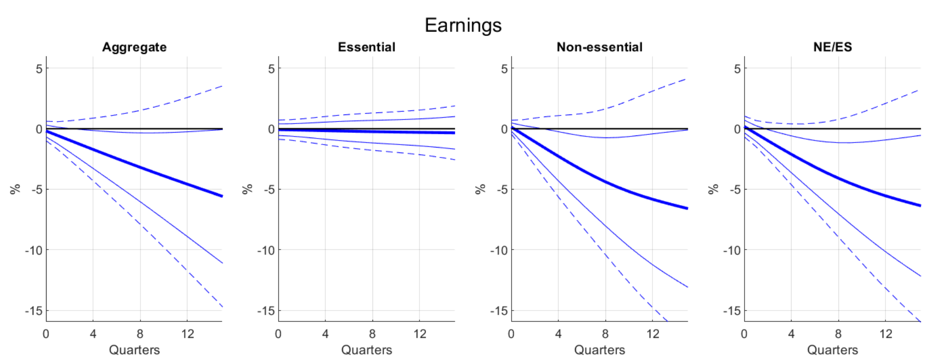
Notes: IRFs estimated by smooth local projections, response to 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument.

Figure F.2: IRFs to contractionary monetary policy shock - Comparison of total earnings series



Notes: IRFs estimated by smooth local projections, response to 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument. The LHS series is the IRF of total compensation of employees (Received: Wage and Salary Disbursements) from the BEA NIPA data. The RHS series is the IRF our constructed equivalent series using CPS data.

Figure F.3: IRFs to contractionary monetary policy shock - Earnings at quarterly frequency

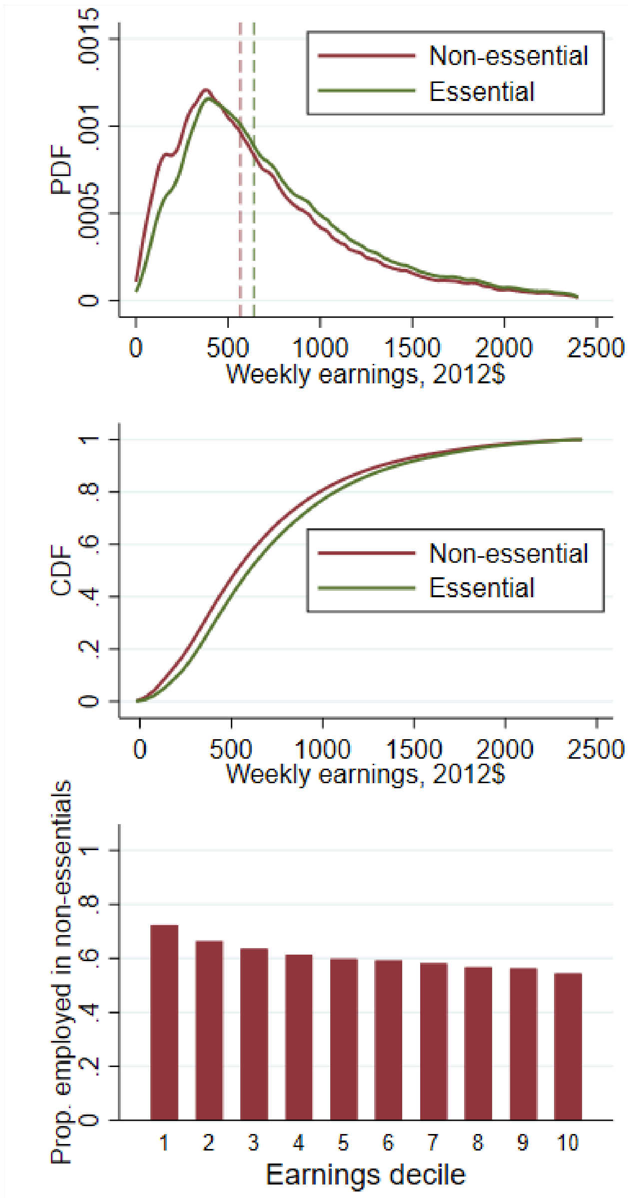


Notes: IRFs estimated by smooth local projections (smooth IRFs) and standard local projections (non-smooth IRFs), response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument, robust to the information effect. Sample and specification as in main text, quarterly frequency data used. 90% and 68% confidence intervals.

F.7 Additional earnings distribution results

Figure F.4 shows the PDF and CDF of earnings distributions, plus the proportion of employment in non-essentials across the earnings deciles. The CDF demonstrates the the the CDF of essential earnings first order stochastically dominates the CDF of non-essential earnings.

Figure F.4: Non-essential and essential - Earnings distribution



Notes: Earnings distributions within essential and non-essential industries. Underlying data is pooled Jan 1982 - December 2020, from the CPS, as described in the text. Panel 1 shows the kernel density plot along the median of each distribution, panel 2 shows the corresponding CDF, and panel 3 shows the percent of employees working in non-essential industries for each decile of the income distribution (deciles computed annually).

F.8 Computing the shares of Hand-to-Mouth workers

This section outlines the method implemented to compute the shares of Hand-to-Mouth (HtM) workers employed in essential and non-essential industries across the income distribution, using data from the Panel Survey of Income Dynamics (PSID). The dataset offers comprehensive information on households' balance sheets and income, along with information on the industry occupations of the households' members. In each wave, a total of 17,280 households are surveyed, and the dataset includes sample weights to ensure that the analysis is representative of the U.S. population.

The first step involves identifying hand-to-mouth households within the sample. Following the approach proposed by Kaplan et al. (2014), we classify households as hand-to-mouth if their liquid assets in a given year are less than their monthly income, which is defined as annual income divided by 12. In the PSID, we define liquid assets as the total amount held in checking and savings accounts, certificates of deposit, T-bills, and bonds, plus the total invested in stocks, stock mutual funds, or investment trusts (excluding stocks held in retirement accounts), minus total liquid debt. Household income is defined as the sum of labor income of the reference person, labor income of the spouse, business income, and government transfers, excluding social security. In Table F.4, we provide detailed information on the specific PSID variables that we use.

For each wave in our sample, households are assigned to income deciles based on the distribution of total households' income from the PSID. Additionally, each household is assigned to either essential or non-essential industries, based on the occupation of the family member with the highest labour income, considering both the reference person and the spouse. Households in which both the reference person and the spouse are retired, disabled, students, or self-employed are excluded. We also exclude the households where the reference person is younger than 22 years old or older than 79 years old, as in Kaplan et al. (2014).

Industries in the PSID are classified according to 3-digit Census codes, allowing for straightforward linkage with our essential and non-essential classification. However, a minor complication arises because industries in the PSID are classified using 2000 Census Codes until 2015 and using 2012 Census Codes from 2017 onward, while our classification is based on 1990 Census Codes. To resolve this issue, we employ industry code crosswalks from the Census Bureau to connect the 2000 and 2012 industry codes to the 1990 industry codes. After classifying each household into HtM groups, income deciles, and industry classifications for each wave of the PSID, it is straightforward to compute a time series of the share of hand-to-mouth households within each income decile and across essential and non-essential industries. Note that throughout the analysis we weight each observation with the sample weights provided in the PSID, which allow the survey sample to be representative of the US population. As the sample restrictions in Kaplan et al. (2014) cause some observations to drop, we rescale the sample weights by a factor given by the sum of weights in the original sample over the sum of weights associated with the observations that we keep, for each wave of the survey.

In Figure 4, we report the average of HtM shares in essential and non-essential sectors across all PSID waves between 2003 and 2021. The start of the sample is dictated by the availability of detailed employment information, which appeared first the PSID wave of 2003. We follow the same procedure outlined in this section to construct shares of hand-to-mouth

Table F.4: Variables definition and code in the PSID

Variable	**PSID CODE**
Total amount in checking and saving accounts, certificates of deposits, T-bills, and bonds	W28 AMT ALL ACCOUNTS until 2017 and W28A AMT CK/SAVING ACCT + W28 AMT CD/BONDS/TB from 2019
Total amount invested in stocks, stock mutual funds, or investment trusts, not including stocks in retirement accounts	IMP VALUE STOCKS (W16)
Liquid debt	IMP VAL CREDIT CARD DEBT (W39A) from 2011 and as IMP VALUE OTH DEBT before 2011, because IMP VAL CREDIT CARD DEBT (W39A) is not available before 2011
Labor income from the reference person plus the labor income of the spouse	LABOR INCOME OF REF PERSON + LABOR INCOME OF SPOUSE
Business income	TOTAL BUSINESS INCOME
Government Transfers	HEAD AND SPOUSE TRANSFER INCOME + TOTAL TRANSFER INCOME OF OFUMS
Family Income	TOTAL FAMILY INCOME
Industry Occupations	BC21 MAIN IND FOR JOB 1 (RP) & DE21 MAIN IND FOR JOB 1 (SP)
Retirement Status, Disabled Status, Student Status	BC1 EMPLOYMENT STATUS-1ST MENTION & DE1 EMPLOYMENT STATUS-1ST MENTION
Self Employment Status	BC22 WORK SELF/OTR?–JOB 1 & DE22 WORK SELF/OTR?–JOB 1
Sample Weight	CORE/IMMIGRANT FAM WEIGHT NUMBER 1

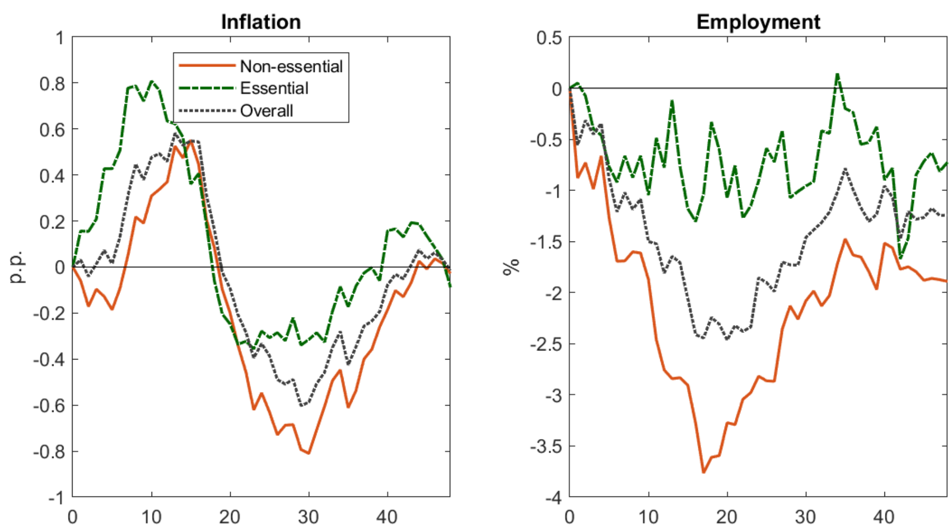
Notes: This table displays the specific variable codes of each variable from the PSID used in our analyisis.

workers along the income distribution by durables and non-durable industries. The results are displayed in Appendix Figure B.3.

F.9 Descriptive statistics and additional charts

To complement Figure 2 in the main text, which shows the dynamics of non-essential and essential consumption and earnings after recessions, Figure F.5 shows the corresponding inflation and employment dynamics. Inflation in the non-essential decelerates more rapidly than in the essential sector, though this heterogeneity is more mild. Here, we focus on core inflation, to remove the more supply-driven dynamics of energy and food inflation[3]. Employment in the non-essential sector sharply contracts, to a peak of nearly 4% below trend in the second year of the recession, while essential earnings decline by only 1%.

Figure F.5: Essentials and Non-essentials over the business cycle - Prices and Employment



Response of essential and non-essential series, starting from the peak of the previous expansion, as defined by NBER. Includes all recession peaks since 1973 where non-essential and essential serie s for each variable are available for a full 48 months after the peak (peaks in 1973m11, 1981m7, 1990m7, 2001m3, 2007m12 and see sample definitions in text). For employment, this shows the cyclical component of the logged variable detrended using the HP filter ($\lambda = 14,440$). Inflation is y/y core inflation, also detrended using the HP filter. All series are normalised to 0 at the inital period by taking the peak observation from all periods.

In Table F.5, we provide descriptive statistics of the constructed essential and non-essential series described above and in the main text. Consumption, employment and median earnings of non-essentials are more volatile than essentials, and covary more with industrial production. In contrast, prices of non-essentials are less volatile and less cyclical (if not mildly countercyclical), a fact we ascribe to the volatility of food and energy prices. In Table F.6, we report the average values of the time series in Figure F.6, which underlies Figure 2.

[3]For the rest of the paper where we analyse identified responses to exogenous monetary policy shocks, this is no longer necessary and we instead address the response of the complete price index.

Table F.5: Descriptive statistics

	Consumption	Prices	Employment	Earnings
Correlation with Industrial Production				
Aggregate	0.68	-0.11	0.74	0.47
Essential	0.52	-0.02	0.36	0.17
Non-essential	0.73	-0.22	0.75	0.51
St. dev relative to Industrial Production				
Aggregate	0.15	0.10	0.16	0.24
Essential	0.14	0.19	0.13	0.22
Non-essential	0.21	0.09	0.21	0.32

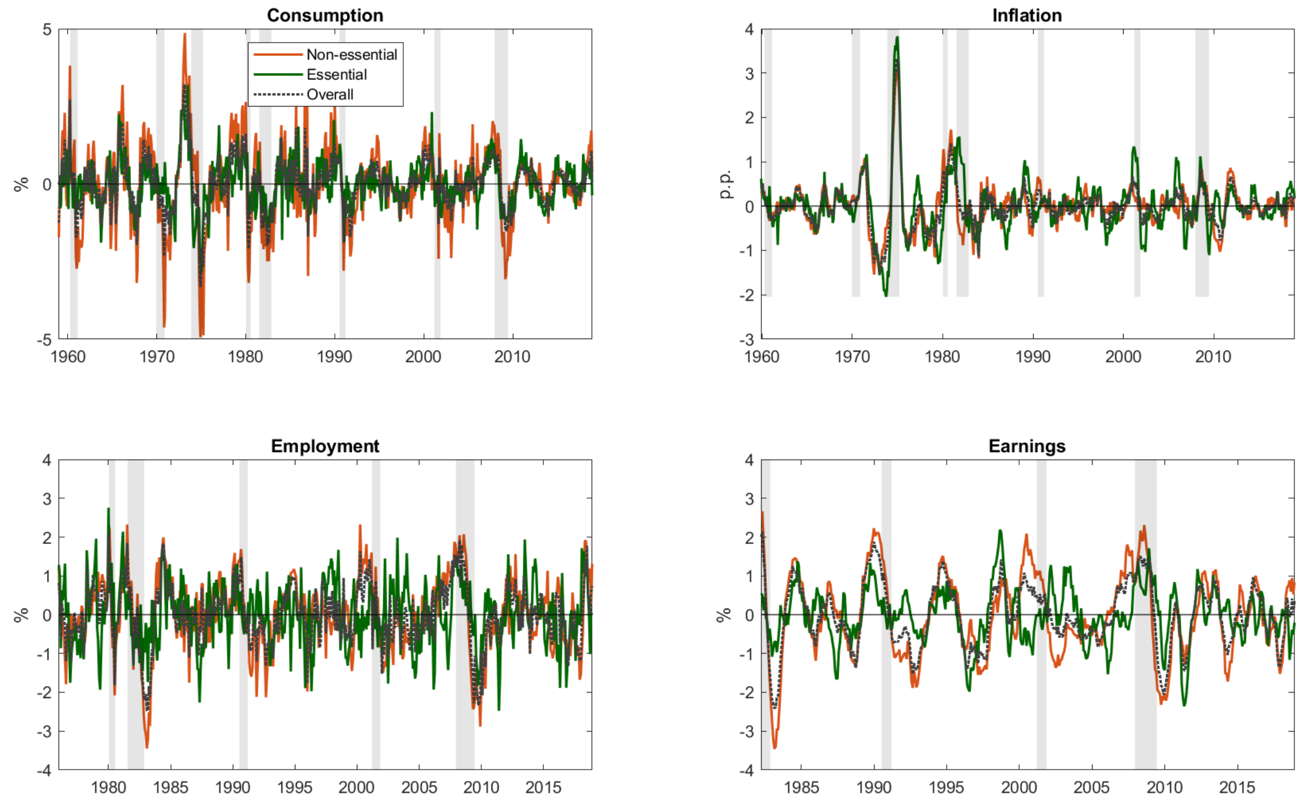
Notes: Descriptive statistics for essentials and non-essentials series. All variables are year-on-year log differences. Panel 1 shows the standard deviation of the series, divided by the standard deviation of industrial production. Panel 2 reports the correlations with industrial production. Monthly data. Sample ends in March 2020 and begins at the earliest available point for each series: January 1960 for consumption and prices, January 1977 for employment and January 1983 for earnings. Price and consumption are based on PCE data and employment and earnings are from CPS data, constructed as described in the text.

Table F.6: Average amount and share, Essentials vs Non-essentials

	Average annual amount			*Share of overall (%)*
	Overall	Essential	Non-essential	Non-essential
Consumption per capita ($)	21,710	10,267	11,443	53%
Employment (millions)	93.4	30.6	62.9	67%
Median labour earnings ($)	31,127	33,025	29,333	94%

Notes: Average annual consumption, employment and median annual wages, in essentials and non-essentials, over the sample period. The final column shows the non-essential consumptiona and employment shares and the non-essential median wage as a % of overall median wages. Only the value of consumption and employment categorised into essentials and non-essentials is included in ‘Overall’, excluding uncategorised. Consumption is per capita chained PCE in 2012$, median wages are deflated to 2012$. Calculations details and data sources are in the text.

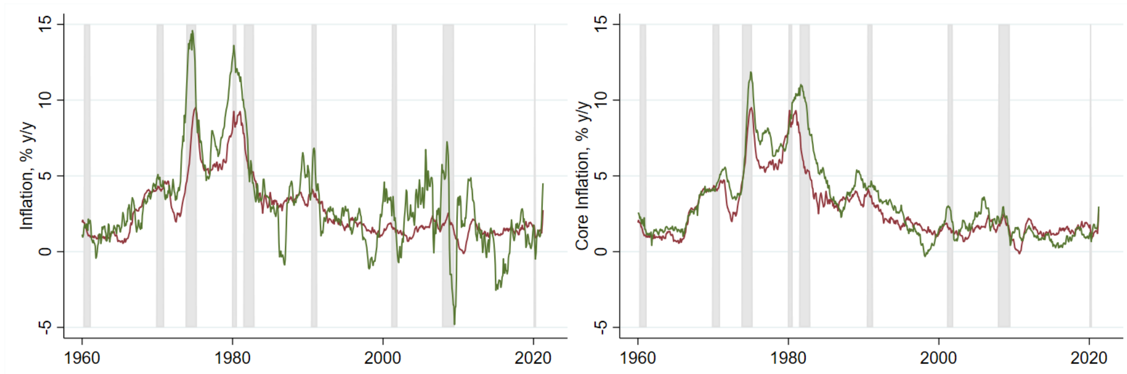
Figure F.6: Essentials and Non-essentials over time



Underlying series of Figure 2. For consumption, employment and earnings, this shows the cyclical component of the logged variable detrended using the HP filter ($\lambda = 14,440$). For earnings, this refers to total earnings and the inital log series is centred 6-month rolling average, to reduce noise. Inflation is y/y core inflation, also detrended using the HP filter.

Core timeseries. Food and energy are essential categories, and may account for much of the variability in the essential price series, where the essential prices are (perhaps unexpectedly) more volatile than the non-essentials. We construct core essential and non-essential series, excluding the same categories as in the aggregate core series from the BEA. Comparison of the timeseries are in Figure F.7. These series are used in Figure F.5.

Figure F.7: Non-essential and essentials inflation - Headline vs Core



Notes: Non-essential and essential time-series inflation, LHS is headline, RHS is core (excluding food and energy). Underlying data sources are the PCE by Type of Product tables from the BEA, described in detail in the text. NBER recession dates shaded.

F.10 State-level analysis methodology

Figure 5 in the main text shows the the correlation between state-level employment changes during recessions and state-level non-essential consumption shares.

To construct this we used:

- Monthly BLS state-level employment data, derived from the CES.
  - We used the raw (non-seasonally adjusted) series, which starts in 1939, and seasonally adjusted it using the X-13ARIMA-SEATS approach. This gives seasonally adjusted state level employment series. These only start in 1973, due to limits on how long the series you can seasonally adjust can be using this procedure (but covers most of our sample period).
  - To identify state-specific recession dates by identifying the state-specific peak and trough of employment within 12 months before/after the NBER recession dates, excluding states where employment did not decline.
- State-level PCE series. The BEA provides these annually for 1997-present. The consumption categories available are somewhat more aggregated than those we are using for our main analysis, so the average non-essential shares do not exactly correspond. Non-essential shares are consumption shares from the BEA's state-level annual PCE series. We average these over the entire sample available for the series shown on the x-axis.

G SLP-IV implementation details

The point estimates for the IRFs for the SLP-IV approach have been estimated using the procedure suggested in Barnichon and Brownlees (2019):

1. We estimate a (standard) first stage by regressing the 1-year yield on the instrument and controls, and extract the predicted values of the endogenous variables $\hat{x}$
2. Use the predicted values in the SLP approach (following the notation in Barnichon and Brownlees (2019)):
   - $\hat{\mathcal{X}}_{\beta,t}$ is a d_t x K matrix where the (h,k)th element is $B_k(h)\hat{x}_t$, and this is stacked with the control matrices in the same way to produce the matrix $\hat{\chi}$.
3. Estimate the second stage SLP by generalised ridge regression: $\hat{\theta} = (\hat{\mathcal{X}}'\hat{\mathcal{X}} + \lambda\mathbf{P})^{-1}\hat{\mathcal{X}}'Y$

λ is selected using a five-fold cross-validation procedure, as suggested by Barnichon and Brownlees. We shrink towards a B-spline of order 2, which shrinks towards a line.

The SLP Newey-West standard errors Barnichon and Brownlees (2019) suggest are:

$$
\begin{aligned}
\widehat{V}(\hat{\theta}) =& T\left[\sum_{t=1}^{T-H_{\min}} \mathcal{X}_t'\mathcal{X}_t + \lambda\mathbf{P}\right]^{-1}\left[\hat{\Gamma}_0 + \sum_{l=1}^{L} w_l\left(\hat{\Gamma}_l + \hat{\Gamma}_l'\right)\right] \\
&\times\left[\sum_{t=1}^{T-H_{\min}} \mathcal{X}_t'\mathcal{X}_t + \lambda\mathbf{P}\right]^{-1}
\end{aligned}
$$

where $w_l = 1 - l/(1+L)$ and $\hat{\Gamma}_l = \frac{1}{T}\sum_{l+1}^{T-H_{\min}} \mathcal{X}_t'\hat{\mathcal{U}}_t\hat{\mathcal{U}}_{t-l}'\ \mathcal{X}_{t-l}$ where $\hat{\mathcal{U}}_t$ are the residuals from the second stage.

To contruct SLP-IV SEs, we use the generated regressor equivalent of this:

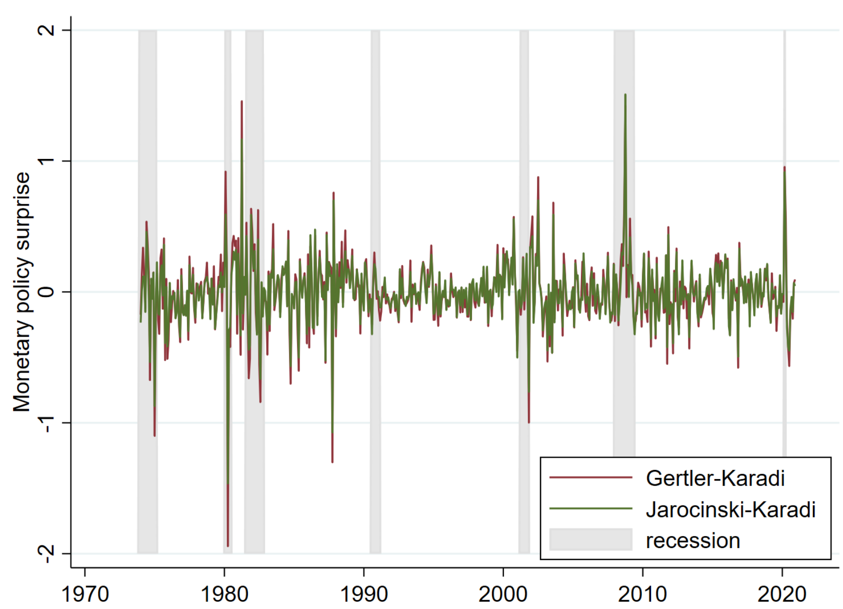
$$
\begin{aligned}
\widehat{V}(\hat{\theta}) =& T\left[\sum_{t=1}^{T-H_{\min}} \hat{\mathcal{X}}_t'\hat{\mathcal{X}}_t + \lambda\mathbf{P}\right]^{-1}\left[\hat{\Gamma}_0 + \sum_{l=1}^{L} w_l\left(\hat{\Gamma}_l + \hat{\Gamma}_l'\right)\right] \\
&\times\left[\sum_{t=1}^{T-H_{\min}} \hat{\mathcal{X}}_t'\hat{\mathcal{X}}_t + \lambda\mathbf{P}\right]^{-1}
\end{aligned}
$$

where $w_l = 1 - l/(1+L)$ and $\hat{\Gamma}_l = \frac{1}{T}\sum_{l+1}^{T-H_{\min}} \hat{\mathcal{X}}_t'\hat{\mathcal{U}}_t\hat{\mathcal{U}}_{t-l}'\ \hat{\mathcal{X}}_{t-l}$. Following Hansen (2021) $\hat{\mathcal{U}}_t = Y - \mathcal{X}\hat{\theta}$ are the residuals used, ie using the controls $\mathcal{X}$ constructed using the actual values of x rather than the first stage predicted values $\hat{x}$. If we set $\lambda = 0$, so no smoothing and penalising the results, this is the same as standard Newey-West standard errors for LP-IV. The autocorrelation lag used is the minimum between the Newey-West (1994) suggestion ($T^{1/4}$) and a linear increase with the estimation horizon. In an omitted robustness check, we also use lag-augmentation, with an extra lag of the controls and White standard errors, which set L=0, so no correction for auto-correlation.

G.1 Monetary policy surprises

Figure G.1 shows the monetary policy surprise series extracted from the proxy-SVAR. The Gertler-Karadi series is the main surprise series used in the empirical section, and Jarocinski-Karadi series is used in Appendix C.3.

Figure G.1: Monetary policy surprise series



Notes: Monetary policy surprises, extracted from a proxy SVAR as described in Section 3.1. The Gertler-Karadi surprises are extracted from a proxy SVAR estimated using the (updated) monetary policy instrument proposed by Gertler and Karadi (2015), while the Jarocinski-Karadi surprises are from using the monetary policy instrument robust to the information effect proposed by Jarocinski and Karadi (2020).

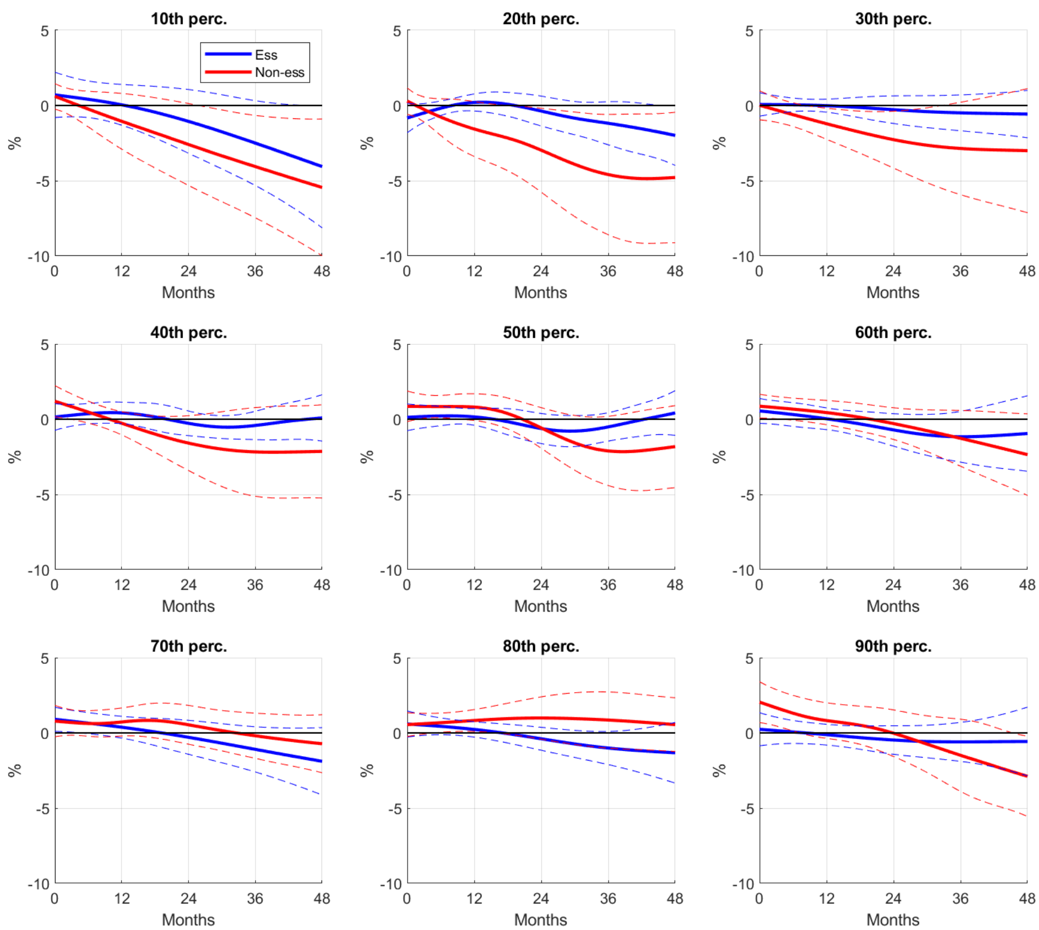
H Additional empirical results and robustness

This section shows some additional empirical and checks that our results are robust to alternative choices of specifications.

H.1 Earnings distribution IRFs

Figure H.1 shows the IRFs of earnings percentiles show in the main text, Figure 7, with their confidence intervals.

Figure H.1: IRFs to contractionary monetary policy shock - Earnings distribution

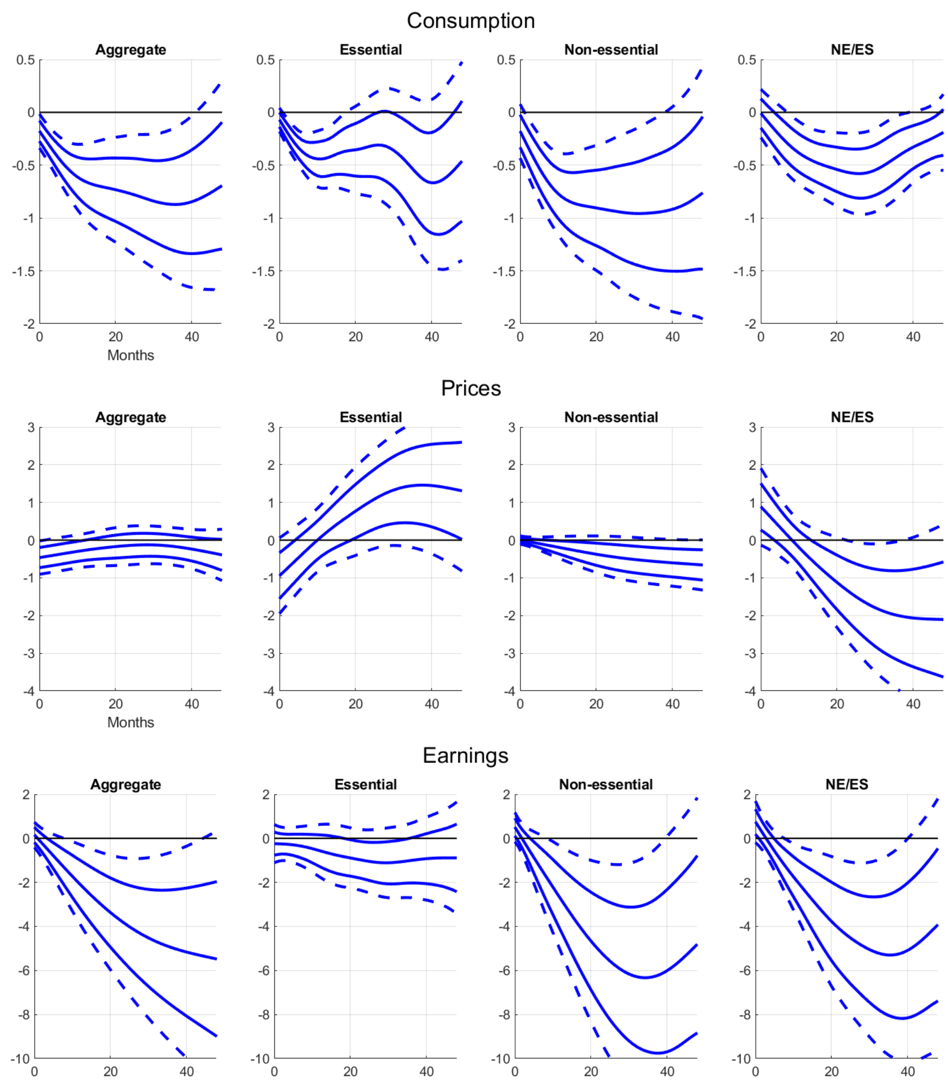


Notes: IRFs estimated by smooth local projections, response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument. 90% confidence intervals displayed. Sample periods and controls are specified in the main text and Appendix C.1.

H.2 Adding COVID to the sample period

In our main sample, we end the estimation period in December 2019. This omits the effects of Covid-19, where non-essentials and essentials responded differently to the shock partly due to sector-specific reductions in activity not directly driven by the mechanism we propose here.[4] To check that our results are robust to adding the effects of the Covid-19 period, we estimate the IRFs for samples ending in December 2020 in Figures H.2 and H.3. The magnitude and degree of heterogeneity in responses is increased with this sample, but in our main results we prefer to focus on the more conservative set of results, excluding Covid, to ensure that only entirely voluntary deferral of non-essential consumption is considered.

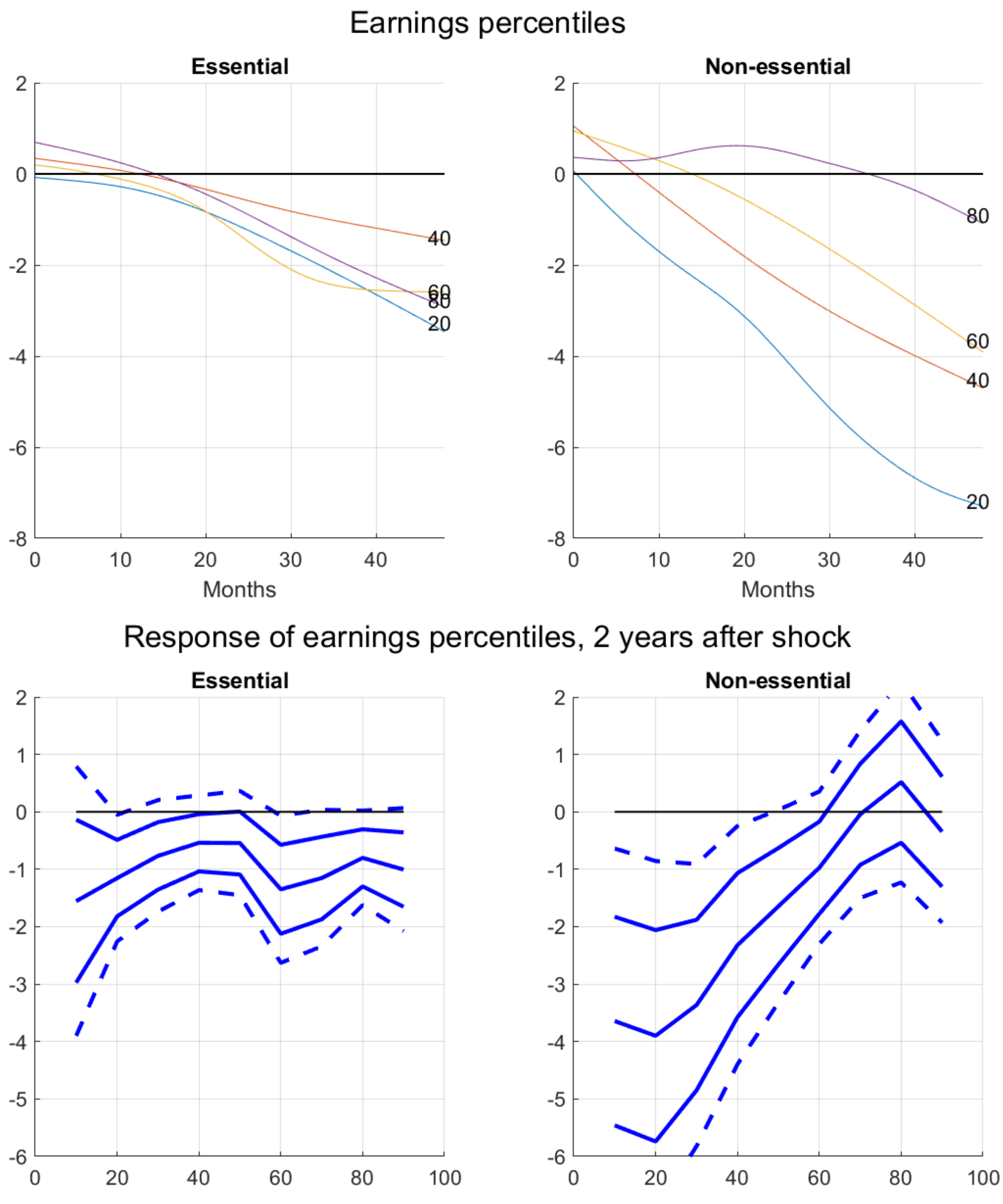
Figure H.2: IRFs to contractionary monetary policy shock - Consumption and Prices



Notes: IRFs estimated by smooth local projections, response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument, robust to the information effect. Sample period ends in December 2020.

[4] We envisage that a main reason for the differential shutdowns across sectors were precisely because certain types of consumption are not intertemporally substitutable, consistent with our mechanism. Our identification strategy of estimating the response to monetary policy shocks should alleviate this concern.

Figure H.3: IRFs to contractionary monetary policy shock - Earnings distribution

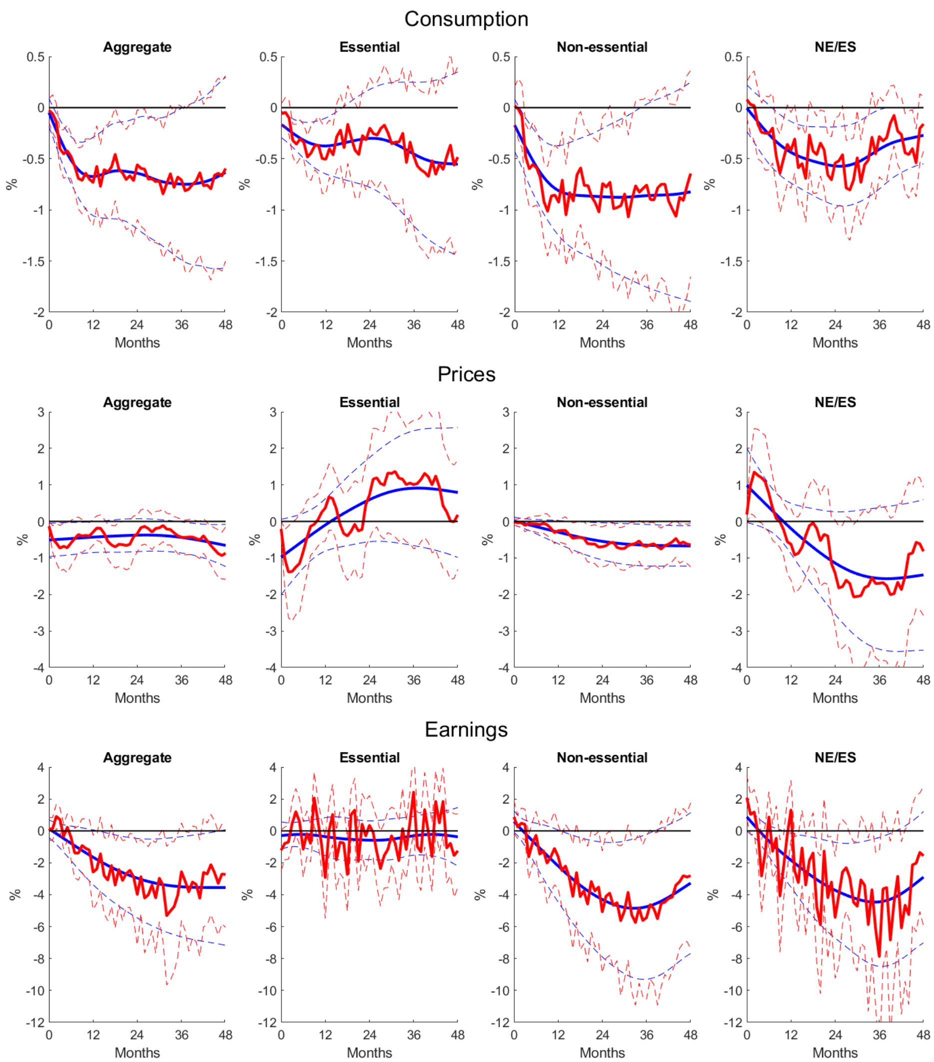


Notes: IRFs estimated by smooth local projections, response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument, robust to the information effect. Sample ends December 2020, otherwise the specification remains in the main body of the text. 68 and 90% confidence intervals displayed.

H.3 IRFs with (unsmoothed) local projections

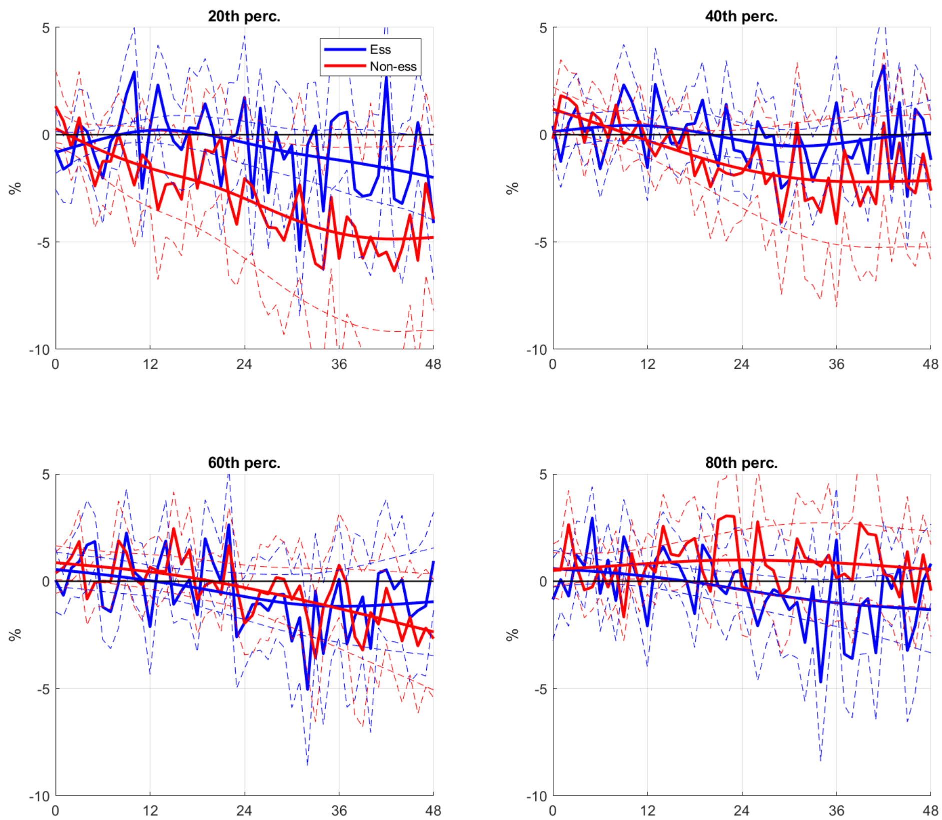
To show that our results are robust to using standard local projections, rather than smoothed local projections, Figures H.4 shows our main results for consumption, prices and earnings are similar for standard LP, but the introduction of smoothing allows us to more clearly see the key results. H.5 shows the IRFs for selected percentiles of the earnings distribution; due to the noise in the earnings series, it is harder to see clear patterns from the LP results.

Figure H.4: IRFs to contractionary monetary policy shock - Consumption, Prices and Earnings



Notes: IRFs estimated by smooth local projections (blue) and standard local projections (red), response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument, robust to the information effect. Samples and specifications as described in the main text.

Figure H.5: IRFs to contractionary monetary policy shock - Earnings distribution



Notes: IRFs estimated by smooth local projections (smooth IRFs) and standard local projections (non-smooth IRFs), response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument, robust to the information effect. Sample and specification as in main text. 90% confidence intervals.

I Model derivations

In this appendix, we provide detailed additional derivations for the theoretical model. The objective is to highlight the solution method, the steady state computation, and the log-linear equilibrium.

I.1 Equilibrium

The competitive equilibrium consists of 31 endogenous allocations $\{C_t$, C_t^E, C_t^N, $C_{H,t}^E$, $C_{H,t}^N$, $C_{L,t}^E$, $C_{L,t}^N$, $C_{H,t,0}^E$, $C_{H,t,0}^N$, $C_{L,t,0}^E$, $C_{L,t,0}^N$, $N_{H,t}$, $N_{L,t}$, $N_{H,t}^E$, $N_{H,t}^N$, $N_{L,t}^E$, $N_{L,t}^N$, $b_{H,t}$, $\zeta_{H,t}$, $\zeta_{L,t}$, $\Pi_{H,t}^r$, $\Pi_{L,t}^r$, $\Pi_t^{r,N}$, $\Pi_t^{r,E}$, Y_t, Y_t^E, Y_t^N, $K_{E,t}^f$, $F_{E,t}^f$, $K_{N,t}^f$, $F_{N,t}^f\}$, 13 prices $\{w_{H,t}$, $w_{L,t}$, π_t, π_t^E, π_t^N, $\pi_{t,Lasp}$, $\pi_{t,Paasche}$, p_t^N, P_t^E, P_t^N, R_t, $\mathcal{S}_t^E$, $\mathcal{S}_t^N\}$, and 3 exogenous processes $\{A_t^E$, A_t^N, $\varepsilon_t^{mp}\}$, with P_0^E normalised to one; such that households, final good producers, retailers, and wholesalers optimise, the central bank follows a Taylor rule, the treasury follows the tax rules, profits are disbursed according to the profit rule, and markets clear. To avoid repetition, we re-write the full set of equations only in the linearised equilibrium.

I.2 Steady state computation

We define a steady state variable simply without the time subscript. We solve for a zero-inflation steady state ($\pi^E = \pi^N = 1$). We set the transfers to the Calvo retailers at $\tau^E = 1/\varepsilon^E$ and $\tau^N = 1/\varepsilon^N$ to ensure no steady state markups ($\mathcal{S}^E = \mathcal{S}^N = 1$) and zero steady state profits. We normalise the steady state price level for the essential good at 1 ($P^E = 1$) and solve for the steady state relative price p^N.

Wages. We solve for wages from the wholesalers problem. As long as $\alpha^E \neq \alpha^N$, the formula is:

$$w_L = (p^N)^{\frac{1-\alpha^E}{\alpha^N-\alpha^E}} \left[\left(A^E (1-\alpha^E)^{1-\alpha^E} (\alpha^E)^{\alpha^E} \right)^{-(1-\alpha^N)} \left(A^N (1-\alpha^N)^{1-\alpha^N} (\alpha^N)^{\alpha^N} \right)^{(1-\alpha^E)} \right]^{\frac{1}{\alpha^N-\alpha^E}} \tag{5}$$

$$w_H = (p^N)^{\frac{-\alpha^E}{\alpha^N-\alpha^E}} \left[\left(A^E (1-\alpha^E)^{1-\alpha^E} (\alpha^E)^{\alpha^E} \right)^{\alpha^N} \left(A^N (1-\alpha^N)^{1-\alpha^N} (\alpha^N)^{\alpha^N} \right)^{-\alpha^E} \right]^{\frac{1}{\alpha^N-\alpha^E}} \tag{6}$$

Consumption. To solve for consumption, first note that in steady state attentive and inattentive consumers all have the same consumption level. Next, plug the labour supply choice, the intra-temporal choice between essential and non-essential goods in the budget constraint and use the zero profit and zero transfer in steady state. This leads for each household $k = \{H, L\}$ to a one non-linear equation in the consumption of essentials:

$$C_k^E + \varphi^{\gamma^N} \left(p^N\right)^{1-\gamma^N} (C_k^E)^{\frac{\gamma^N}{\gamma^E}} = w_k^{1+\frac{1}{\chi}} \xi^{-\frac{1}{\chi}} (C_k^E)^{-\frac{1}{\chi\gamma^E}} \quad k = \{H, L\} \tag{7}$$

With non-homotheticity, this equation cannot be solved analytically, but can be solved easily numerically.

Algorithm to find the steady state. For a given set of structural parameters, we compute the steady state with the following algorithm. Vary p^N such that we compute:

1. w_H, and w_L analytically with (5) and (6).
2. C_H^E and C_L^E numerically with (7).
3. C_H^N, C_L^N, N_H, N_L from the household/union problem.
4. Y^E and Y^N from the goods market clearing conditions.
5. N_H^E and N_L^N from firms' labour demand functions.
6. The difference between $N_H^E + N_H^N$ and $\mu_H N_H$.

Iterate on p^N until the difference it is zero. Alternatively, the last step can be substituted with the difference between $N_L^E + N_L^N$ and $\mu_L N_L$ by Walras law (one market clearing condition can be ignored).

In each estimation draw, we target the steady state consumption shares of Ricardian and hand-to-mouth agents of non-essentials: $\bar{C}_H^N \equiv \frac{p^N C_H^N}{p^N C_H^N + C_H^E}$ and $\bar{C}_L^N \equiv \frac{p^N C_L^N}{p^N C_L^N + C_L^E}$. To do so, we vary the relative preference parameter for non-essentials φ and the relative productivity of the two sectors $a^E \equiv A^E/A^N$. φ affects the average consumption share. a^E affects the relative wage, and, therefore, the relative consumption shares, thanks to the non-homotheticity in the utility function.

I.3 Log-linear equilibrium

We solve the log-linearised model. Steps are standard, we log-linearise each variable, except for profits, which we linearise as they are zero in steady state. Log-linearised and linearised variables are hatted. The only feature to note is that all CPI inflation indices simplify to the same steady states weighted average of inflation:

$$\hat{\pi}_t = \hat{\pi}_{t,Lasp} = \hat{\pi}_{t,Paasche} = \frac{C^E}{C^E + p^N C^N}\hat{\pi}_t^E + \frac{p^N C^N}{C^E + p^N C^N}\hat{\pi}_t^N$$

Equilibrium. The competitive equilibrium consists of 27 endogenous allocations $\{\hat{C}_t$, $\hat{C}_t^E$, $\hat{C}_t^N$, $\hat{C}_{H,t}^E$, $\hat{C}_{H,t}^N$, $\hat{C}_{L,t}^E$, $\hat{C}_{L,t}^N$, $\hat{C}_{H,t,0}^E$, $\hat{C}_{H,t,0}^N$, $\hat{C}_{L,t,0}^E$, $\hat{C}_{L,t,0}^N$, $\hat{N}_{H,t}$, $\hat{N}_{L,t}$, $\hat{N}_{H,t}^E$, $\hat{N}_{H,t}^N$, $\hat{N}_{L,t}^E$, $\hat{N}_{L,t}^N$, $\hat{\zeta}_{H,t}$, $\hat{\zeta}_{L,t}$, $\hat{\Pi}_{L,t}^r$, $\hat{\Pi}_t^{r,N}$, $\hat{\Pi}_t^{r,E}$, $\hat{Y}_t$, $\hat{Y}_t^E$, $\hat{Y}_t^N$, $E\hat{arn}_t^E$ $E\hat{arn}_t^N\}$, 9 prices $\{\hat{w}_{H,t}$, $\hat{w}_{L,t}$, $\hat{\pi}_t$, $\hat{\pi}_t^E$, $\hat{\pi}_t^N$, $\hat{p}_t^N$, $\hat{R}_t$, $\hat{\mathcal{S}}_t^E$, $\hat{\mathcal{S}}_t^N\}$, and 3 exogenous processes $\{\hat{A}_t^E$, $\hat{A}_t^N$, $\varepsilon_t^{mp}\}$; such that households, final good producers, retailers, and wholesalers optimise, the central bank follows a Taylor rule, the treasury follows the tax rule, profits are disbursed according to the profit rule, and markets clear. The equilibrium is characterised by the following equations:

$$-\frac{1}{\gamma^E}\hat{C}_{H,t,0}^E + \frac{1}{\gamma^N}\hat{C}_{H,t,0}^N = -\hat{p}_t^N$$

$$\frac{1}{\gamma^E}\mathbb{E}_t\left(\hat{C}_{H,t+1,0}^E\right) = \frac{1}{\gamma^E}\hat{C}_{H,t,0}^E - \mathbb{E}_t(\hat{\pi}_{t+1}^E) + \hat{R}_t$$

$$\hat{C}^E_{H,t} = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j}\left(\hat{C}^E_{H,t,0}\right)$$

$$\hat{C}^N_{H,t} = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j}\left(\hat{C}^N_{H,t,0}\right)$$

$$-\frac{1}{\gamma^E}\hat{C}^E_{L,t,0} + \frac{1}{\gamma^N}\hat{C}^N_{L,t,0} = -\hat{p}^N_t$$

$$C^E_L \hat{C}^E_{L,t} + p^N C^N_L(\hat{p}^N_t + \hat{C}^N_{L,t}) = w_L N_L(\hat{w}_{L,t} + \hat{N}_{L,t}) + \frac{\hat{\Pi}^r_{L,t}}{\mu_L}$$

$$\hat{C}^E_{L,t} = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j}\left(\hat{C}^E_{L,t,0}\right)$$

$$\hat{C}^N_{L,t} = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j}\left(\hat{C}^N_{L,t,0}\right)$$

$$\hat{\zeta}_{H,t} = -\frac{1}{\gamma^E}\hat{C}^E_{H,t}(1-\bar{C}^N_H) - \left(\frac{1}{\gamma^N}\hat{C}^N_{H,t} + \hat{p}^N_t\right)\bar{C}^N_H$$

$$\hat{\zeta}_{L,t} = -\frac{1}{\gamma^E}\hat{C}^E_{L,t}(1-\bar{C}^N_L) - \left(\frac{1}{\gamma^N}\hat{C}^N_{L,t} + \hat{p}^N_t\right)\bar{C}^N_L$$

$$\chi\hat{N}_{H,t} - \hat{\zeta}_{H,t} = \hat{w}_{H,t}$$

$$\chi\hat{N}_{L,t} - \hat{\zeta}_{L,t} = \hat{w}_{L,t}$$

$$\hat{\pi}^N_t = \beta\mathbb{E}_t(\hat{\pi}^N_{t+1}) + \kappa^N \hat{\mathcal{S}}^N_t$$

$$\hat{\pi}^E_t = \beta\mathbb{E}_t(\hat{\pi}^E_{t+1}) + \kappa^E \hat{\mathcal{S}}^E_t$$

$$\pi^N_t = \pi^E_t + p^N_t - p^N_{t-1}$$

$$\hat{Y}^N_t = \hat{A}^N_t + \alpha^N \hat{N}^N_{L,t} + (1-\alpha^N)\hat{N}^N_{H,t}$$

$$\hat{\mathcal{S}}^N_t + \hat{Y}^N_t - \hat{N}^N_{H,t} = \hat{w}_{H,t} - \hat{p}^N_t$$

$$\hat{\mathcal{S}}^N_t + \hat{Y}^N_t - \hat{N}^N_{L,t} = \hat{w}_{L,t} - \hat{p}^N_t$$

$$\hat{Y}^E_t = \hat{A}^E_t + \alpha^E \hat{N}^E_{L,t} + (1-\alpha^E)\hat{N}^E_{H,t}$$

$$\hat{\mathcal{S}}^E_t + \hat{Y}^E_t - \hat{N}^E_{H,t} = \hat{w}_{H,t}$$

$$\hat{\mathcal{S}}^E_t + \hat{Y}^E_t - \hat{N}^E_{L,t} = \hat{w}_{L,t}$$

$$N^E_H \hat{N}^E_{H,t} + N^N_H \hat{N}^N_{H,t} = \mu_H N_H \hat{N}_{H,t}$$

$$N^E_L \hat{N}^E_{L,t} + N^N_L \hat{N}^N_{L,t} = \mu_L N_L \hat{N}_{L,t}$$

$$\hat{\pi}_t = \frac{C^E}{C^E + p^N C^N}\hat{\pi}^E_t + \frac{p^N C^N}{C^E + p^N C^N}\hat{\pi}^N_t$$

$$Y\hat{Y}_t = Y^E \hat{Y}^E_t + p^N Y^N \hat{Y}^N_t$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\phi_\pi\left(\mathbb{E}_t(\hat{\pi}_{t+1})\right) + \phi_Y \hat{Y}_t\right) + \sigma^{mp}\varepsilon^{mp}_t$$

$$\hat{\Pi}^r_{L,t} = \phi^E_{\Pi,L}\hat{\Pi}^{r,E}_t + \phi^N_{\Pi,L}\hat{\Pi}^{r,N}_t$$

$$\hat{\Pi}^{r,E}_t = -Y^E \hat{\mathcal{S}}^E_t$$

$$
\begin{aligned}
\hat{\Pi}_t^{r,N} &= -Y^N p^N \hat{\mathcal{S}}_t^N \\
C^E \hat{C}_t^E &= \mu_H C_H^E \hat{C}_{H,t}^E + \mu_L C_L^E \hat{C}_{L,t}^E \\
C^N \hat{C}_t^N &= \mu_H C_H^N \hat{C}_{H,t}^N + \mu_L C_L^N \hat{C}_{L,t}^N \\
\hat{Y}_t^E &= \hat{C}_t^E \\
\hat{Y}_t^N &= \hat{C}_t^N \\
\hat{Earn}_t^E &= \frac{w_H N_H^E}{w_H N_H^E + w_L N_L^E}(\hat{w}_{H,t} + \hat{N}_{H,t}^E) + \frac{w_L N_L^E}{w_H N_H^E + w_L N_L^E}(\hat{w}_{L,t} + \hat{N}_{L,t}^E) \\
\hat{Earn}_t^N &= \frac{w_H N_H^N}{w_H N_H^N + w_L N_L^N}(\hat{w}_{H,t} + \hat{N}_{H,t}^N) + \frac{w_L N_L^N}{w_H N_H^N + w_L N_L^N}(\hat{w}_{L,t} + \hat{N}_{L,t}^N)
\end{aligned}
$$

Notice that the equilibrium conditions include four equations with an infinite sum of past expectations (the mapping from each inattentive consumer consumption to the family wide one). To solve the model with a state space representation, we adopt a method proposed by Verona and Wolters (2014) for sticky expectations models. We solve for a truncated set of past expectations. The key insight is that, if we care only about IRFs, as we do here (our estimation uses IRF matching), we can truncate the expectations at the horizon of the IRFs and have no loss in precision (say in period 16). $\mathbb{E}_{t-j}(\hat{C}_{H,t,0}^E)$ will be zero for each $j > 16$, that is before the shock happens.

J Model estimation and counterfactual

In this appendix, we present the estimation procedure, the full set of estimated IRFs, and the details of our counterfactual exercise.

J.1 Estimation

We estimate the model with a limited-information Bayesian approach, that is, with a impulse response matching with a maximum a posteriori (MAP) estimation procedure. We follow the estimation procedure of Mertens and Ravn (2011), with the weighting matrix choice of Guerron-Quintana et al. (2017), extended to a MAP setting. Given our model, we estimate a vector of parameters Θ_2 (the parameters in Panel A of Table 1) conditional on a vector of calibrated parameters Θ_1 (the parameters in Panel B of Table 1). The quasi-likelihood:

$$F(\hat{\Lambda}_d|\Theta_2,\Theta_1) = \left(\frac{1}{2\pi}\right)^{\frac{T}{2}} |\Sigma_d| \exp\left[-\frac{1}{2}\left(\hat{\Lambda}_d - \Lambda(\Theta_2|\Theta_1)\right)' \Sigma_d^{-1} \left(\hat{\Lambda}_d - \Lambda(\Theta_2|\Theta_1)\right)\right]$$

This maps the difference in the estimated IRFs with smooth local projections $\hat{\Lambda}_d$ to the model based IRFs $\Lambda(\Theta_2|\Theta_1)$. We stack the IRFs in a vector of dimension T, in the baseline setting equal to 112 (16 quarters times 7 variables). As weighting matrix, we follow Guerron-Quintana et al. (2017) and use a diagonal matrix with the squared standard errors from the smooth local projection estimates for each IRF element. We denote $p(\Theta_2)$ the prior distribution over the estimated parameters. We follow the common procedure of imposing bounds in the prior draws, but none bind at the estimated values. The quasi-posterior:

$$F(\Theta_2|\hat{\Lambda}_d,\Theta_1) \propto F(\hat{\Lambda}_d|\Theta_2,\Theta_1)p(\Theta_2)$$

Maximum a posterior estimation maximises the posterior over estimated parameters. The practical benefit, over frequentist impulse response matching matching, is that it allows to incorporate priors over parameters.

$$\hat{\Theta}_2 = \arg\max_{\Theta_2} F(\Theta_2|\hat{\Lambda}_d,\Theta_1)$$

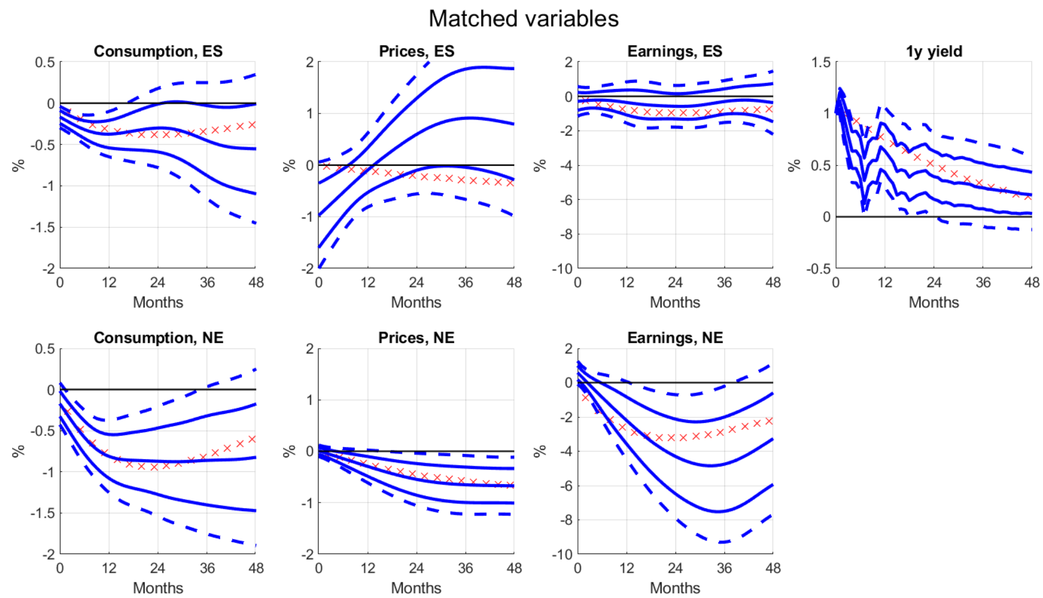
We compute the standard errors of $\hat{\Theta}_2$ with the delta method. The formula for the asymptotic covariance matrix, from Mertens and Ravn (2011):

$$\begin{aligned}
\Sigma_{\Theta_2} &= \Lambda_{\Theta_2} \frac{\partial \Lambda(\Theta_2|\Theta_1)'}{\partial \Theta_2} \Sigma_d^{-1} \Sigma_S \Sigma_d^{-1} \frac{\partial \Lambda(\Theta_2|\Theta_1)}{\partial \Theta_2} \Lambda_{\Theta_2} \\
\Lambda_{\Theta_2} &\equiv \left[\frac{\partial \Lambda(\Theta_2|\Theta_1)'}{\partial \Theta_2} \Sigma_d^{-1} \frac{\partial \Lambda(\Theta_2|\Theta_1)}{\partial \Theta_2}\right]^{-1} \\
\Sigma_S &\equiv \Sigma_d + \Sigma_m
\end{aligned}$$

Where we use Σ_d in the last line, following Guerron-Quintana et al. (2017). Notice that we use the model based IRFs, not the IRFs estimated on data simulated from the model as Mertens and Ravn (2013) do, so that $\Sigma_m = 0$ and the overall expression for the parameters covariance matrix simplifies.

Estimated IRFs. Figure J.1 shows the empirical IRFs in blue and the estimated IRFs in red for the whole set of matched variables. We estimate the end-quarter impulse response for these variables, as described in the main text. For consumption, earnings and prices we match the estimated IRFs for non-essentials and essentials using our SLP empirical approach. For 1y yields, we estimate the impulse response from the proxy-SVAR.

Figure J.1: IRFs to contractionary monetary policy shock - Matched variables from model



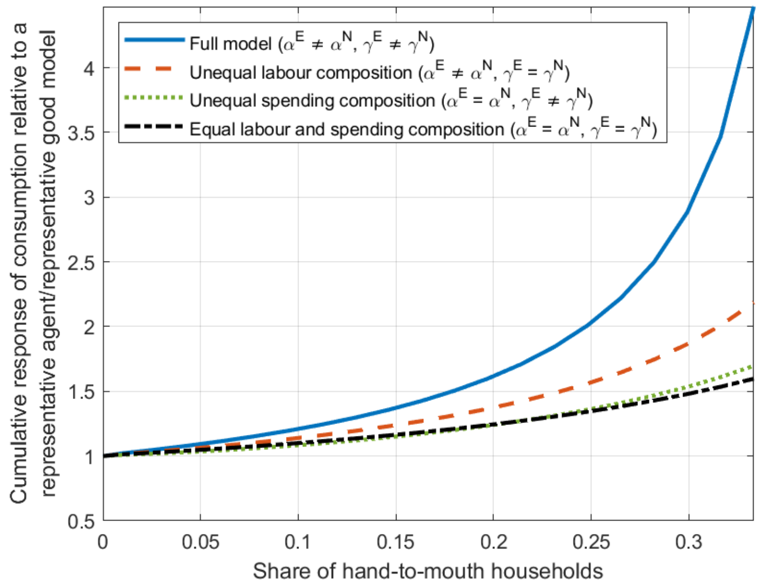
Notes: Consumption, prices, earnings: IRFs estimated by smooth local projections, response to a 100bp increase in 1y yields, instrumented using monetary policy shocks derived from Gertler and Karadi (2015) high-frequency identified monetary policy instrument. Sample periods and controls are specified in the main text. Interest rate: estimated using Proxy-SVAR, as described in text.

J.2 Counterfactual

A key difference between the representative agent cases and the heterogeneous agents models is the presence of hand-to-mouth consumers only in the latter. This feature interacts with both cyclical product demand composition and cyclical labour demand composition to further amplify the effects of monetary policy. To illustrate this triple interaction, in Figure J.2, we report the aggregate consumption response in the four heterogeneous agents cases of Table 2 as we vary the share of hand-to-mouth households, μ^L, from 0 to 0.33, a value consistent with the empirical literature on estimating MPCs (e.g. Johnson et al., 2006).[5]

[5] To ensure that the economic significance of hand-to-mouth agents reflects their relative size, for any value of μ^L, we adjust the labour income shares accrued to hand-to-mouth households in Figure J.2 such that $\alpha^J = \bar{\alpha}^J \frac{\mu^L}{\bar{\mu}^L}$, where $\bar{\mu}^L, \bar{\alpha}^J$ are the values taken by these parameters in the estimated full structural model.

Figure J.2: On the Sources of Amplification



Notes: Amplification is measured by the cumulative IRF of consumption of each model, divided by the cumulative response of consumption in the restricted model with no hand-to-mouth agents. The figure depicts four scenarios: (i) the unrestricted full model as blue solid line, (ii) unequal labour sectoral composition (i.e. $\gamma^E = \gamma^N$) as orange dashed line, (iii) unequal spending composition (i.e. $\alpha^E = \alpha^N$) as green dotted line, and (iv) equal labour and equal spending composition (i.e. $\alpha^E = \alpha^N$ and $\gamma^E = \gamma^N$) as black broken line. The latter is often referred to in the literature as Two-Agents New-Keynesian (TANK) model. As in Table 2, whenever $\alpha^E = \alpha^N = \tilde{\alpha}$, we set $\tilde{\alpha}$ so as to match the relative steady state labour earnings across the two agents. Whenever $\gamma^E = \gamma^N$, we set the IES to equal the average IES in the estimated full structural model.

In each simulation, the cumulated consumption response to monetary policy is normalized by the cumulated effect in the representative agent/good case. This implies that each point of Figure J.2 can be interpreted as the extent of amplification of that model (and for that value of μ^L) relative to the representative benchmark. The blue line refers to the full structural model that features both cyclical product demand composition and cyclical labour demand composition, whereas the black broken line summarizes the results of the restricted model with neither of the two. The dashed orange line and the dotted green line stand for the two intermediate cases of only unequal labour composition or only unequal spending composition, respectively.

Four main results emerge from this exercise. First, in all models, a higher share of hand-to-mouth consumers leads to a monotonic increase in the extent of amplification, though the nonlinearity of this relationship is very heterogeneous across models. Second, the case with both equal labour composition and equal spending composition, often referred to as Two-Agents New-Keynesian (TANK) model, exhibits a degree of amplification relative to the representative agent/representative that is between 15% and 50%, over the empirically plausible range of [0.15, 0.33] for the average MPC, consistent with the evidence in earlier studies on U.S. data such as Patterson (2023) and Bilbiie et al. (2023). Third, non-homothetic preferences seem to add little amplification over TANK, whereas the marginal contribution of the unequal labour sectoral composition appears relatively larger. Fourth, the extent of amplification in the full model (depicted as blue line) is consistently larger than the sum of the dashed orange line and the green dotted line over the whole range of values for μ^L. This reveals that the triple interaction between cyclical product demand composition, cyclical labour demand composition and hand-to-mouth households generates a strong complementarity that greatly amplifies business-cycle fluctuations relative not only to the representative agent/representative good case but also to heterogeneous agents models that only feature the double interaction between constrained agents and heterogeneity in either consumers' spending or workers' sectoral composition.

In Table 2, we compared the cumulative response of aggregate consumption in counterfactual exercises. In this appendix, we complete this analysis by showing the dis-aggregated consumption responses by different goods.

Table J.1 shows the cumulative IRFs of non-essential and essential consumption between the non-homothetic and homothetic representative agent counterfactuals. As seen in Table 2, aggregate consumption responds equally in both cases. However, unlike for aggregates, non-homotheticity does change sectoral outcomes. Notice that our irrelevance result of Appendix Section K demonstrates the irrelevance of sectoral heterogeneities for aggregates in the representative agent setting (albeit for a simpler model than that used in the counterfactuals). This table demonstrates numerically the same result applies for the representative agent model used in the counterfactuals.

Table J.1: Counterfactuals of Essentials and Non-essentials in RANK

	Representative Agent		
	C	C^E	C^N
Homothetic	1.00	1.00	1.00
Non-Homothetic	1.00	0.34	1.54

Notes: Each cell display the ratio of the cumulative IRF of the counterfactual experiment over the cumulative IRF of the representative agent model with homothetic preferences evaluated at the estimated model parameters. The first columns shows aggregate consumption, the second essential consumption, and the third non-essential consumption. In the homothetic case, we set the IES equal to the estimated average IES in the baseline model.

K The Analytics of non-homotheticity in RANK

In this appendix, we present the a proof on when non-homotheticity does not amplify business cycles. We show that the non-homothetic RANK has the same response to monetary policy of aggregate variables then a homothetic RANK with the IES equal to the IES of the non-homothetic RANK. This implies that non-homotheticity does not matter per-se for amplification, but it matters only when interacts with other features, as labour market heterogeneity, finanacial constraints, price stickiness, heterogeneous capital intensities, etc. We formalize this idea with Proposition 2 and Corollary 1.

Proposition 2 *Consider a simplified version of the model of Sections 4 and D. Take an attentive representative agent version with non-homothetic utility (2) and a simplified Taylor rule of the form $R_t = \phi_\pi \mathbb{E}_t(\pi_{t+1}) + \varepsilon_t^{mp}$. The impact of the monetary policy shock on total consumption is characterised by the average intertemporal elasticity of substitution and on CPI inflation by the average intertemporal elasticity of substitution and the slope of the Phillips curves:*

$$\frac{\partial \hat{C}_t}{\partial \varepsilon_t^{mp}} = -(\underbrace{\gamma^E(1-\bar{C}^N) + \gamma^N \bar{C}^N}_{Average\ IES})$$

$$\frac{\partial \hat{\pi}_t}{\partial \varepsilon_t^{mp}} = -\underbrace{\kappa}_{Slope\ of\ NKPC}(1 + \underbrace{\gamma^E(1-\bar{C}^N) + \gamma^N \bar{C}^N}_{Average\ IES})$$

Corollary 1 *Consider a simplified version of the model of Sections 4 and D. Take an attentive representative agent version with homothetic utility*

$$U(C_t^E, C_t^N, N_t) = \frac{(C_t^E)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \varphi \frac{(C_t^N)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} - \xi \frac{N_t^{1+\chi}}{1+\chi}$$

such that the intertemporal elasticity of substitution γ is equal to $\gamma^E(1-\bar{C}^N) + \gamma^N \bar{C}^N$ of the model presented in Proposition 2, and a simplified Taylor rule of the form $R_t = \phi_\pi \mathbb{E}_t(\pi_{t+1}) + \varepsilon_t^{mp}$. The impact of the monetary policy shock on total consumption is characterised by the intertemporal elasticity of substitution and on CPI inflation by the intertemporal elasticity of substitution and the slope of the Phillips curves:

$$\frac{\partial \hat{C}_t}{\partial \varepsilon_t^{mp}} = -\underbrace{\gamma}_{IES}$$

$$\frac{\partial \hat{\pi}_t}{\partial \varepsilon_t^{mp}} = -\underbrace{\kappa}_{Slope\ of\ NKPC}(1 + \underbrace{\gamma}_{IES})$$

We now move to prove both statements. The intuition of the result is that relative prices are a state variable but they do not respond to an aggregate shock in the representative agent model. In addition, the two New-Keynesian Phillips curves have the same expressions for the map from aggregate consumption to overall inflation.

Proof of Proposition 2.

We solve analytically the model which features non-homothetic preferences with a representative agent who is attentive. Operationally, we set $\alpha^N = \alpha^E = 0$ as we have one agent only. We set $\lambda = 1$. We as have only one agent, we have $C_{H,t} = C_t$ and similarly for sector specific variables and employment variables. We can rewrite the first set of equilibrium conditions:

$$
\begin{aligned}
\hat{p}_t^N &= \frac{1}{\gamma^E}\hat{C}_t^E - \frac{1}{\gamma^N}\hat{C}_t^N \\
\hat{N}_t + \frac{1}{\gamma^E}\hat{C}_t^E &= \hat{w}_t \\
\hat{Y}_t^N &= \hat{N}_t^N \\
\hat{\mathcal{S}}_t^N &= \hat{w}_t - \hat{p}_t^N \\
\hat{Y}_t^E &= \hat{N}_t^E \\
\hat{\mathcal{S}}_t^E &= \hat{w}_t \\
\hat{Y}_t^N &= \hat{C}_t^N \\
\hat{Y}_t^E &= \hat{C}_t^E \\
\hat{C}_t &= (1-\bar{C}^N)\hat{C}_t^E + \bar{C}^N\hat{C}_t^N \\
\hat{N}_t &= (1-\bar{C}^N)\hat{N}_t^E + \bar{C}^N\hat{N}_t^N
\end{aligned}
$$

We can solve this systems to express $\hat{\mathcal{S}}_t^N$ and $\hat{\mathcal{S}}_t^N$ as function of $\hat{C}_t$ and $\hat{p}_t^N$:

$$
\begin{bmatrix} \hat{\mathcal{S}}_t^E \\ \hat{\mathcal{S}}_t^N \end{bmatrix} = \begin{bmatrix} \frac{1+\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} & \frac{\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} \\ \frac{1+\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} & \frac{\gamma^E(1-\bar{C}^N)}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} \end{bmatrix} \begin{bmatrix} \hat{C}_t \\ \hat{p}_t^N \end{bmatrix}
$$

Compactly:

$$
\begin{bmatrix} \hat{S}_t^E \\ \hat{S}_t^N \end{bmatrix} = \begin{bmatrix} a_C^{SE} & a_p^{SE} \\ a_C^{SN} & a_p^{SN} \end{bmatrix} \begin{bmatrix} \hat{C}_t \\ \hat{p}_t^N \end{bmatrix}
$$

Next, we map goods specific consumption and inflation to their aggregate counterparts. First, express consumption of essentials as a function of overall consumption and relative prices with the overall consumption definition and the intra-termprial consumption good choice.

$$
\begin{aligned}
\hat{C}_t &= (1-\bar{C}^N)\hat{C}_t^E + \bar{C}^N\hat{C}_t^N \\
\hat{C}_t &= (\gamma^E(1-\bar{C}^N) + \gamma^N\bar{C}^N)\frac{1}{\gamma^E}\hat{C}_t^E - \gamma^N\bar{C}^N\hat{p}_t^N
\end{aligned}
$$

We can express inflation in essential and non-essential as function of overall inflation and relative prices with the mapping between relative prices and the inflation rates:

$$
\begin{aligned}
\hat{\pi}_t &= (1-\bar{C}^N)\hat{\pi}_t^E + \bar{C}^N\hat{\pi}_t^N \\
\hat{\pi}_t^N &= \hat{\pi}_t + (1-\bar{C}^N)(\hat{p}_t^N - \hat{p}_{t-1}^N)
\end{aligned}
$$

and symmetrically:

$$\hat{\pi}_t^E = \hat{\pi}_t - \bar{C}^N(\hat{p}_t^N - \hat{p}_{t-1}^N)$$

We can now turn to the inter-temporal part of the model. The equations are:

$$\begin{aligned}
\hat{\pi}_t^E &= \beta \mathbb{E}_t(\hat{\pi}_{t+1}^E) + \kappa \hat{\mathcal{S}}_t^E \\
\hat{\pi}_t^N &= \beta \mathbb{E}_t(\hat{\pi}_{t+1}^N) + \kappa \hat{\mathcal{S}}_t^N \\
\frac{1}{\gamma^E}\mathbb{E}_t\left(\hat{C}_{t+1}^E\right) &= \frac{1}{\gamma^E}\hat{C}_t^E - \mathbb{E}_t(\hat{\pi}_{t+1}^E) + \hat{R}_t
\end{aligned}$$

Substitute-in the mappings from inflation in essential and non-essentials and essential consumption to overall consumption, inflation, and relative prices.

$$\begin{aligned}
\hat{\pi}_t - \bar{C}^N(\hat{p}_t^N - \hat{p}_{t-1}^N) &= \beta \mathbb{E}_t(\hat{\pi}_{t+1}) - \beta \bar{C}^N(\mathbb{E}_t(\hat{p}_{t+1}^N) - \hat{p}_t^N) + \kappa \hat{\mathcal{S}}_t^E \\
\hat{\pi}_t + (1-\bar{C}^N)(\hat{p}_t^N - \hat{p}_{t-1}^N) &= \beta \mathbb{E}_t(\hat{\pi}_{t+1}) + \beta(1-\bar{C}^N)(\mathbb{E}_t(\hat{p}_{t+1}^N) - \hat{p}_t^N) + \kappa \hat{\mathcal{S}}_t^N \\
\frac{1}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}\mathbb{E}_t\left(\hat{C}_{t+1}\right) + \frac{\bar{C}^N(1-\bar{C}^N)(\gamma^N-\gamma^E)}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}\mathbb{E}_t\left(\hat{p}_{t+1}^N\right) &= \\
= \frac{1}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}\hat{C}_t + \frac{\bar{C}^N(1-\bar{C}^N)(\gamma^N-\gamma^E)}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}\hat{p}_t^N &- \mathbb{E}_t(\hat{\pi}_{t+1}) + \hat{R}_t
\end{aligned}$$

We can substitute in a simplified Taylor rule: $\hat{R}_t = \phi_\pi \mathbb{E}(\pi_{t+1}) + \varepsilon_t^{mp}$ and the expressions that map responses of consumption and relative prices to marginal costs and write the system in matrix form. In the final system the only parameter or convolutions that matter are: γ^E, γ^N, β, κ, $\bar{C}^N$.

$$\begin{bmatrix} 0 & \beta & -\beta\bar{C}^N \\ 0 & \beta & \beta(1-\bar{C}^N) \\ \frac{1}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} & \phi_\pi - 1 & \frac{\bar{C}^N(1-\bar{C}^N)(\gamma^N-\gamma^E)}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} \end{bmatrix} \begin{bmatrix} \mathbb{E}_t(\hat{C}_{t+1}) \\ \mathbb{E}_t(\hat{\pi}_{t+1}) \\ \mathbb{E}_t(\hat{p}_{t+1}^N) \end{bmatrix} +$$

$$\begin{bmatrix} \kappa\frac{1+\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} & -1 & \bar{C}^N(\beta+1) + \kappa\frac{\bar{C}^N\gamma^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} \\ \kappa\frac{1+\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} & -1 & -(1-\bar{C}^N)(\beta+1) - \kappa\frac{(1-\bar{C}^N)\gamma^E}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} \\ -\frac{1}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} & 0 & -\frac{\bar{C}^N(1-\bar{C}^N)(\gamma^N-\gamma^E)}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N} \end{bmatrix} \begin{bmatrix} \hat{C}_t \\ \hat{\pi}_t \\ \hat{p}_t^N \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & -\bar{C}^N \\ 0 & 0 & (1-\bar{C}^N) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{p}_{t-1}^N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \varepsilon_t^{mp} = 0$$

$$A\mathbb{E}(X_{t+1}) + BX_t + CX_{t-1} + H\varepsilon_t^{mp} = 0$$

We solve this system in the case of iid monetary policy shock. We solve it with the undetermined coefficient method. The solution depends on the monetary policy shock and on the state variable, the relative price in the previous period $\hat{p}^N_{t-1}$:

$$\begin{bmatrix} \hat{C}_t \\ \hat{\pi}_t \\ \hat{p}^N_t \end{bmatrix} = \begin{bmatrix} e_1\hat{p}^N_{t-1} + d_1\varepsilon^{mp}_t \\ e_2\hat{p}^N_{t-1} + d_2\varepsilon^{mp}_t \\ e_3\hat{p}^N_{t-1} + d_3\varepsilon^{mp}_t \end{bmatrix} = \begin{bmatrix} e_1 & d_1 \\ e_2 & d_2 \\ e_3 & d_3 \end{bmatrix} \begin{bmatrix} \hat{p}^N_{t-1} \\ \varepsilon^{mp}_t \end{bmatrix}$$

The system with the solution plugged in becomes:

$$A\mathbb{E}(X_{t+1}) + BX_t + CX_{t-1} + H\varepsilon^{mp}_t = 0$$

$$A \begin{bmatrix} e_1(e_3\hat{p}^N_{t-1} + d_3\varepsilon^{mp}_t) \\ e_2(e_3\hat{p}^N_{t-1} + d_3\varepsilon^{mp}_t) \\ e_3(e_3\hat{p}^N_{t-1} + d_3\varepsilon^{mp}_t) \end{bmatrix} + B \begin{bmatrix} e_1\hat{p}^N_{t-1} + d_1\varepsilon^{mp}_t \\ e_2\hat{p}^N_{t-1} + d_2\varepsilon^{mp}_t \\ e_3\hat{p}^N_{t-1} + d_3\varepsilon^{mp}_t \end{bmatrix} + C \begin{bmatrix} 0 \\ 0 \\ \hat{p}^N_{t-1} \end{bmatrix} + H\varepsilon^{mp}_t = 0$$

This creates two sets of systems of equations to solve for, from the coefficients associated with the state variable and with the monetary policy shock:

$$Ae_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + C \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$
$$Ad_3 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + B \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} + H = 0$$

This would be daunting to solve analytically if monetary policy affected the relative price d_3. However, we show that the solution has $d_3 = 0$ by guessing it and verifying it. The uniqueness of the solution is guaranteed by the Taylor principle $\phi_\pi > 1$. The key idea is that the responses of consumption and inflation to the monetary policy shock depend on the average IES only $\gamma^E(1-\bar{C}^N) + \gamma^N\bar{C}^N$ and not on its elements separately. Moreover, the two NKPC display the same terms for inflation and consumption. If this was not the case, say due to labour market heterogeneity or price stickiness heterogeneity, the proof would not go through, showing that non-homotheticity matters only in conjunction with other relevant heterogeneity for aggregate fluctuation.

Guess $d_3 = 0$, then:

$$A0 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + B \begin{bmatrix} d_1 \\ d_2 \\ 0 \end{bmatrix} + H = 0$$
$$B \begin{bmatrix} d_1 \\ d_2 \\ 0 \end{bmatrix} + H = 0$$
$$\begin{bmatrix} \kappa\frac{1+\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}d_1 - d_2 = 0 \\ \kappa\frac{1+\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}d_1 - d_2 = 0 \\ -\frac{1}{\gamma^E(1-\bar{C}^N)+\gamma^N\bar{C}^N}d_1 - 1 = 0 \end{bmatrix}$$

$$d_1 = -(\gamma^E(1-\bar{C}^N) + \gamma^N \bar{C}^N)$$
$$d_2 = -\kappa(1 + \gamma^E(1-\bar{C}^N) + \gamma^N \bar{C}^N)$$

That is consumption responds by the average IES to a monetary policy shock and inflation responds by the Phillips curve slope times by one plus the average IES. This concludes the proof that in a non-homothetic RANK, only the average IES matters for aggregate fluctuations. This concludes the proof. ■

We now move to the corollary: the non-homothetic RANK responses of aggregate variables to monetary policy are the same to a homothetic-RANK with the same average IES.

Proof of Corollary 1. This is immediate, substitute $\gamma = \gamma^E(1-\bar{C}^N) + \gamma^N \bar{C}^N$ for γ^E and γ^N. The system becomes:

$$\begin{bmatrix} 0 & \beta & -\beta\bar{C}^N \\ 0 & \beta & \beta(1-\bar{C}^N) \\ \frac{1}{\gamma} & \phi_\pi - 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t(\hat{C}_{t+1}) \\ \mathbb{E}_t(\hat{\pi}_{t+1}) \\ \mathbb{E}_t(\hat{p}^N_{t+1}) \end{bmatrix} +$$
$$\begin{bmatrix} \kappa\frac{1+\gamma}{\gamma} & -1 & \bar{C}^N(\beta+1+\kappa) \\ \kappa\frac{1+\gamma}{\gamma} & -1 & -(1-\bar{C}^N)(\beta+1+\kappa) \\ -\frac{1}{\gamma} & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_t \\ \hat{\pi}_t \\ \hat{p}^N_t \end{bmatrix} +$$
$$\begin{bmatrix} 0 & 0 & -\bar{C}^N \\ 0 & 0 & (1-\bar{C}^N) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{p}^N_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \varepsilon^{mp}_t = 0$$

The proof goes through in the same way, with the solution to a monetary policy shock being:

$$d_1 = -\gamma$$
$$d_2 = -\kappa(1+\gamma)$$
$$d_3 = 0$$

This concludes the proof. ■

The same result would go through also with more complicated models, as long as non-homotheticity does not interact directly with other heterogeneity. It would go through with inattentiveness or persistent monetary policy. We showed this numerically in Table J.1.

L Unconventional fiscal policy

In this appendix, we describe the equilibrium of the version of the model with unconventional fiscal policy with heterogeneous goods and heterogeneous households. The set-up of the model is described in Appendix L.

Equilibrium. The competitive equilibrium consists of 34 endogenous allocations $\{\hat{C}_t$, $\hat{C}^E_t$, $\hat{C}^N_t$, $\hat{C}^E_{H,t}$, $\hat{C}^N_{H,t}$, $\hat{C}^E_{L,t}$, $\hat{C}^N_{L,t}$, $\hat{C}^E_{H,t,0}$, $\hat{C}^N_{H,t,0}$, $\hat{C}^E_{L,t,0}$, $\hat{C}^N_{L,t,0}$, $\hat{N}_{H,t}$, $\hat{N}_{L,t}$, $\hat{N}^E_{H,t}$, $\hat{N}^N_{H,t}$, $\hat{N}^E_{L,t}$, $\hat{N}^N_{L,t}$, $\hat{\zeta}_{H,t}$, $\hat{\zeta}_{L,t}$, $\hat{\Pi}^r_{L,t}$, $\hat{\Pi}^{r,N}_t$, $\hat{\Pi}^{r,E}_t$, $\hat{Y}_t$, $\hat{Y}^E_t$, $\hat{Y}^N_t$, $\hat{Earn}^E_t$, $\hat{Earn}^N_t$, $\hat{\tau}^{VAT}_t$, $\hat{\tau}^{VATN}_t$, $\hat{\tau}^{VATE}_t$, $\hat{\tau}^{Payroll}_{H,t}$, $\hat{\tau}^{Payroll}_{L,t}$, $\hat{t}_{H,t}$, $\hat{t}_{L,t}\}$, 9 prices $\{\hat{w}_{H,t}$, $\hat{w}_{L,t}$, $\hat{\pi}_t$, $\hat{\pi}^E_t$, $\hat{\pi}^N_t$, $\hat{p}^N_t$, $\hat{R}_t$, $\hat{\mathcal{S}}^E_t$, $\hat{\mathcal{S}}^N_t\}$, and 6 exogenous processes $\{\hat{A}^E_t$, $\hat{A}^N_t$, ε^{VAT}_t, ε^{VATE}_t, ε^{VATN}_t, $\varepsilon^{mp}_t\}$; such that households, final good producers, retailers, and wholesalers optimise, the central bank follows a Taylor rule, the treasury follows the tax rules, profits are disbursed according to the profit rule, and markets clear. The equilibrium is characterised by the following equations:

$$-\frac{1}{\gamma^E}\hat{C}^E_{H,t,0} + \frac{1}{\gamma^N}\hat{C}^N_{H,t,0} = -\hat{p}^N_t - \hat{\tau}^{VATN}_t + \hat{\tau}^{VATE}_t$$

$$\frac{1}{\gamma^E}\mathbb{E}_t\left(\hat{C}^E_{H,t+1,0}\right) = \frac{1}{\gamma^E}\hat{C}^E_{H,t,0} - \mathbb{E}_t(\hat{\pi}^E_{t+1}) + \hat{R}_t + \\ + \hat{\tau}^{VATE}_t - \mathbb{E}_t(\hat{\tau}^{VATE}_{t+1}) + \hat{\tau}^{VAT}_t - \mathbb{E}_t(\hat{\tau}^{VAT}_{t+1})$$

$$\hat{C}^E_{H,t} = \lambda\sum_{j=0}^{\infty}(1-\lambda)^j\mathbb{E}_{t-j}\left(\hat{C}^E_{H,t,0}\right)$$

$$\hat{C}^N_{H,t} = \lambda\sum_{j=0}^{\infty}(1-\lambda)^j\mathbb{E}_{t-j}\left(\hat{C}^N_{H,t,0}\right)$$

$$-\frac{1}{\gamma^E}\hat{C}^E_{L,t,0} + \frac{1}{\gamma^N}\hat{C}^N_{L,t,0} = -\hat{p}^N_t - \hat{\tau}^{VATN}_t + \hat{\tau}^{VATE}_t$$

$$w_L N_L(\hat{w}_{L,t} + \hat{N}_{L,t} - \hat{\tau}^{Payroll}_{L,t}) = -\frac{\hat{\Pi}^r_{L,t}}{\mu_L} - \frac{\hat{t}_{L,t}}{\mu_L} + C^E_L(\hat{C}^E_{L,t} + \hat{\tau}^{VAT}_t + \hat{\tau}^{VATE}_t) + \\ + p^N C^N_L(\hat{p}^N_t + \hat{C}^N_{L,t} + \hat{\tau}^{VAT}_t + \hat{\tau}^{VATN}_t)$$

$$\hat{C}^E_{L,t} = \lambda\sum_{j=0}^{\infty}(1-\lambda)^j\mathbb{E}_{t-j}\left(\hat{C}^E_{L,t,0}\right)$$

$$\hat{C}^N_{L,t} = \lambda\sum_{j=0}^{\infty}(1-\lambda)^j\mathbb{E}_{t-j}\left(\hat{C}^N_{L,t,0}\right)$$

$$\hat{\zeta}_{H,t} = -\left(\frac{1}{\gamma^E}\hat{C}^E_{H,t} + \hat{\tau}^{VAT}_t + \hat{\tau}^{VATE}_t\right)(1-\bar{C}^N_H) - \\ - \left(\frac{1}{\gamma^N}\hat{C}^N_{H,t} + \hat{p}^N_t + \hat{\tau}^{VAT}_t + \hat{\tau}^{VATN}_t\right)\bar{C}^N_H$$

$$\hat{\zeta}_{L,t} = -\left(\frac{1}{\gamma^E}\hat{C}^E_{L,t} + \hat{\tau}^{VAT}_t + \hat{\tau}^{VATE}_t\right)(1-\bar{C}^N_L) -$$

$$
\begin{aligned}
&- \left(\frac{1}{\gamma^N} \hat{C}^N_{L,t} + \hat{p}^N_t + \hat{\tau}^{VAT}_t + \hat{\tau}^{VATN}_t \right) \bar{C}^N_L \\
\chi \hat{N}_{H,t} - \hat{\zeta}_{H,t} &= \hat{w}_{H,t} - \hat{\tau}^{Payroll}_{H,t} \\
\chi \hat{N}_{L,t} - \hat{\zeta}_{L,t} &= \hat{w}_{L,t} - \hat{\tau}^{Payroll}_{H,t} \\
\hat{\pi}^N_t &= \beta \mathbb{E}_t(\hat{\pi}^N_{t+1}) + \kappa^N \hat{\mathcal{S}}^N_t \\
\hat{\pi}^E_t &= \beta \mathbb{E}_t(\hat{\pi}^E_{t+1}) + \kappa^E \hat{\mathcal{S}}^E_t \\
\pi^N_t &= \pi^E_t + p^N_t - p^N_{t-1} \\
\hat{Y}^N_t &= \hat{A}^N_t + \alpha^N \hat{N}^N_{L,t} + (1-\alpha^N) \hat{N}^N_{H,t} \\
\hat{\mathcal{S}}^N_t + \hat{Y}^N_t - \hat{N}^N_{H,t} &= \hat{w}_{H,t} - \hat{p}^N_t \\
\hat{\mathcal{S}}^N_t + \hat{Y}^N_t - \hat{N}^N_{L,t} &= \hat{w}_{L,t} - \hat{p}^N_t \\
\hat{Y}^E_t &= \hat{A}^E_t + \alpha^E \hat{N}^E_{L,t} + (1-\alpha^E) \hat{N}^E_{H,t} \\
\hat{\mathcal{S}}^E_t + \hat{Y}^E_t - \hat{N}^E_{H,t} &= \hat{w}_{H,t} \\
\hat{\mathcal{S}}^E_t + \hat{Y}^E_t - \hat{N}^E_{L,t} &= \hat{w}_{L,t} \\
N^E_H \hat{N}^E_{H,t} + N^N_H \hat{N}^N_{H,t} &= \mu_H N_H \hat{N}_{H,t} \\
N^E_L \hat{N}^E_{L,t} + N^N_L \hat{N}^N_{L,t} &= \mu_L N_L \hat{N}_{L,t} \\
\hat{\pi}_t &= \frac{C^E}{C^E + p^N C^N} \hat{\pi}^E_t + \frac{p^N C^N}{C^E + p^N C^N} \hat{\pi}^N_t \\
Y \hat{Y}_t &= Y^E \hat{Y}^E_t + p^N Y^N \hat{Y}^N_t \\
\hat{R}_t &= \rho_R \hat{R}_{t-1} + (1-\rho_R) \left(\phi_\pi \left(\mathbb{E}_t(\hat{\pi}_{t+1}) \right) + \phi_Y \hat{Y}_t \right) + \sigma^{mp} \varepsilon^{mp}_t \\
\hat{\Pi}^r_{L,t} &= \phi^E_{\Pi,L} \hat{\Pi}^{r,E}_t + \phi^N_{\Pi,L} \hat{\Pi}^{r,N}_t \\
\hat{\Pi}^{r,E}_t &= -Y^E \hat{\mathcal{S}}^E_t \\
\hat{\Pi}^{r,N}_t &= -Y^N p^N \hat{\mathcal{S}}^N_t \\
C^E \hat{C}^E_t &= \mu_H C^E_H \hat{C}^E_{H,t} + \mu_L C^E_L \hat{C}^E_{L,t} \\
C^N \hat{C}^N_t &= \mu_H C^N_H \hat{C}^N_{H,t} + \mu_L C^N_L \hat{C}^N_{L,t} \\
\hat{Y}^E_t &= \hat{C}^E_t \\
\hat{Y}^N_t &= \hat{C}^N_t \\
E\hat{arn}^E_t &= \frac{w_H N^E_H}{w_H N^E_H + w_L N^E_L} (\hat{w}_{H,t} + \hat{N}^E_{H,t}) + \frac{w_L N^E_L}{w_H N^E_H + w_L N^E_L} (\hat{w}_{L,t} + \hat{N}^E_{L,t}) \\
E\hat{arn}^N_t &= \frac{w_H N^N_H}{w_H N^N_H + w_L N^N_L} (\hat{w}_{H,t} + \hat{N}^N_{H,t}) + \frac{w_L N^N_L}{w_H N^N_H + w_L N^N_L} (\hat{w}_{L,t} + \hat{N}^N_{L,t}) \\
\hat{\tau}^{VAT}_t &= \rho^{VAT} \hat{\tau}^{VAT}_{t-1} + \sigma^{VAT} \varepsilon^{VAT}_t \\
\hat{\tau}^{VATE}_t &= \rho^{VATE} \hat{\tau}^{VATE}_{t-1} + \sigma^{VATE} \varepsilon^{VATE}_t \\
\hat{\tau}^{VATN}_t &= \rho^{VATN} \hat{\tau}^{VATN}_{t-1} + \sigma^{VATN} \varepsilon^{VATN}_t \\
-\hat{\tau}^{Payroll}_{H,t} &= \hat{\tau}^{VAT}_t + (1-\bar{C}^N_H) \hat{\tau}^{VATE}_t + \bar{C}^N_H \hat{\tau}^{VATN}_t \\
-\hat{\tau}^{Payroll}_{L,t} &= \hat{\tau}^{VAT}_t + (1-\bar{C}^N_L) \hat{\tau}^{VATE}_t + \bar{C}^N_L \hat{\tau}^{VATN}_t
\end{aligned}
$$

$$\frac{\hat{t}_{H,t}}{\mu_L} = C_H^E(\hat{\tau}_t^{VAT} + \hat{\tau}_t^{VATE}) + p^N C_H^N(\hat{\tau}_t^{VAT} + \hat{\tau}_t^{VATN}) + w_H N_H(\hat{\tau}_{H,t}^{Payroll})$$

$$\frac{\hat{t}_{L,t}}{\mu_L} = C_L^E(\hat{\tau}_t^{VAT} + \hat{\tau}_t^{VATE}) + p^N C_L^N(\hat{\tau}_t^{VAT} + \hat{\tau}_t^{VATN}) + w_L N_L(\hat{\tau}_{L,t}^{Payroll})$$