Online Appendices for Monetary Policy and the Maturity Structure of Public Debt

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C Data Construction

C.1 Public Debt Data

I construct a monthly dataset of public nominal marketable debt promises at market prices for each future month, that is, the value, today, of the government promises for j months ahead from bond level data. Following Hall and Sargent (2011) methodology, the budget constraint of a government which can issue nominal and inflation linked debt at different maturities reads:

$$\frac{1}{p_t}\sum_{j=0}^{n-1} q_{t,j} d_{t-1,j+1} + \sum_{j=0}^{n-1} \bar{q}_{t,j} \bar{d}_{t-1,j+1} = PS_t + \frac{1}{p_t}\sum_{j=1}^{n} q_{t,j} d_{t,j} + \sum_{j=1}^{n} \bar{q}_{t,j} \bar{d}_{t,j}$$

Where $q_{t,j}$ is the amount of nominal currency in period t that one needs to purchase one unit of nominal currency in period $t+j$, $d_{t,j}$ is the amount of these promises that the government has in period t to pay in period $t+j$, $\bar{q}_{t,j}$ is the amount of period t goods that one needs to purchase one unit of period $t+j$ goods, $\bar{d}_{t,j}$ is the corresponding amount, n is the maximum maturity of public deb, PS_t is the primary surplus in period t goods. For both nominal and inflation-linked debt, as governments issue bonds which are not zero coupon, I strip the coupons and create the equivalent series for the marketable part of $d_{t,j}$ in each month for future promises also dated monthly. With respect to the prices, I use the continuously compounded zero coupon nominal and real yield curve data and compute the zero coupon prices with $q_{t,j} = e^{-\frac{j}{12} r_{t,j}}$ and $\bar{q}_{t,j} = e^{-\frac{j}{12} \bar{r}_{t,j}}$ for the real yield curve. j is divided by 12 to convert it to annual frequency and $r_{t,j}$ is the appropriate interest rate for the nominal yield curve and $\bar{r}_{t,j}$ is the appropriate interest rate for the real yield curve.

For the US, I use bond level CRSP data as Hall and Sargent (2011) at a monthly frequency to compute the quantity series and I use the parameters from Gürkaynak, Sack and Wright (2007) to compute the yield curve. For the UK, I use the quantity data from Ellison and Scott (2020) and official Bank of England yield curve data. As the original yield curve data is at frequencies lower than monthly in the UK data, I interpolate the series within, moreover I assume a constant rate for maturities longer than the ones present in the original data, as I need to price consols. I construct the same series for inflation linked debt from the same sources.

Macaulay duration measures the percent decrease in the market value of this debt following an infinitesimal change in interest rates uniformly across the yield curve $dr = dr_{t,j}$, for $j > 0$. On the other hand, duration-to-GDP measures the same change but not in percent terms but in percentage points of GDP, thereby capturing the overall insurance amount relevant for debt management.

$$MacDur_t = -\frac{\frac{\partial\left(\sum_{j=1}^{n} q_{t,j} d_{t,j}\right)}{\partial r}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}} = -\frac{\frac{\partial\left(\sum_{j=1}^{n} e^{-\frac{j}{12} r_{t,j}} d_{t,j}\right)}{\partial r}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}} = \frac{\sum_{j=1}^{n} \frac{j}{12} q_{t,j} d_{t,j}}{\sum_{j=1}^{n} q_{t,j} d_{t,j}}$$

$$DurGDP_t = -\frac{\frac{\partial\left(\sum_{j=1}^{n} q_{t,j} d_{t,j}\right)}{\partial r}}{GDP_t} = -\frac{\frac{\partial\left(\sum_{j=1}^{n} e^{-\frac{j}{12} r_{t,j}} d_{t,j}\right)}{\partial r}}{GDP_t} = \frac{\sum_{j=0}^{\infty} \frac{j}{12} q_{t,j} d_{t,j}}{GDP_t}$$

The analysis is based on marketable debt, but this should not be a big concern as Hall and Sargent (2011), Ellison and Scott (2020) show that in the period I consider, most of US and UK public debt was marketable, the last periods in which non marketable debt plaid an important role as during the World Wars and during the Korean War for the US. Moreover, I exclude treasury bills which are not present in the data sources I use. This is not problematic as these are very short maturity debt which would get a small weight in the duration-to-GDP measure, as an example a 3 month promise is weighted by 0.25 in duration-to-GDP, whereas a 10 years promise is weighted by 10. These caveats show another advantage of the duration-to-GDP measure compared with Macaulay duration, whereas they are likely to be minor issues for duration-to-GDP and we know that all time series points are lower bonds on the true duration-to-GDP, the same cannot be said for the Macaulay duration. In duration-to-GDP we divide by GDP, which is not affected by issues with measurement of debt, on the other hand Macaulay duration is, as the denominator is computed with debt data as well, so that the exclusion of treasury bills would become problematic.

In quarterly regressions, I divide nominal debt by nominal GDP for that quarter, however this is not feasible for the monthly regressions. There, I need to have a measure of GDP which does not depend on future GDP in order to not create spurious regressions, so interpolation is not an implementable path. In the baseline specification, the nominal income measure is a random walk forecast for nominal GDP, that is for the last month in a quarter I use nominal GDP of that quarter[45], for the other two months I use the nominal GDP for the last quarter[46].

[45] This is appropriate as all debt data is dated at the last day of the month.

[46] In a robustness check on the main empirical results presented on Figure 2 and available upon request, I employ an alternative nominal income measure which is present at monthly frequency: industrial production multiplied by the CPI index. This specification also yields similar results.

US Data Description. Figure 1 in the main text presents the Macaulay duration and the duration-to-GDP for marketable nominal public debt held by the public for the US. In this section, I present similar data with alternative assumptions and with different cuts of the data.

First of all, Figure C.1 presents the nominal marketable public debt over GDP held by the public. This figure can be seen as the ratio between duration-to-GDP over Macaulay duration presented in Figure 1. If we compare the behavior of public debt over GDP with duration-to-GDP, we can see how they correlate, but not one to one, duration-to-GDP increased much faster in the mid eighties and declined much less than public debt in the early aughts. Following we the increase in public debt due to the financial crisis we see a flattening of public debt, but duration-to-GDP keeps raising due to the lengthening in maturities (this holds even after netting out FED holdings).

The left panel of Figure C.2 present how duration-to-GDP looks with alternative measures. The baseline blue solid line shows the data as Figure 1, it is the measure constructed from marketable public debt held by the public at market value. The red long-dashed line *face value* shows the same measure with debt at face value, that is, each debt promise $d_{t,j}$ is not deflated with the yield curve data $q_{t,j}$, but is simply multiplied by one. This is the measure used in Greenwood and Vayanos (2014). We can see how this measures overvalues periods with relatively high interest rates due to discounting, but the overall path is smoother than when we deflated with yield curve data. The green short-dashed line *also FED holdings* plots the same market value measure, but with the overall outstanding amount and not only the amount held by the public. The additional debt includes intra-agency holdings and especially FED holding of public debt. We can see how the green line was always higher than than the blue one, but before QE2, the amount was constant. Following the start of QE2, the FED held a significant amount of long public debt. We can quantify the amount of interest rate risk held by the FED with duration-to-GDP of government held debt, to almost one percent of GDP in the mid twenty tens. Finally, the sienna dashed line *also TIPS* includes to the baseline series also inflation-linked bonds (TIPS). As duration-to-GDP measures how much would the value of public debt decrease following a one percent increase in interest rate across the nominal yield curve, we need to make an additional assumption on how changes in the nominal yield curve translates into the real yield curve. As this would put an additional assumption on the baseline measures and as TIPS are a relatively small share of public debt, especially in the estimation period, I did not include them in the baseline measure. However, for completeness I add them here and show in Appendix D that results are robust to their inclusion. Consistent with yield curve responses following monetary policy shocks highlighted by Nakamura and Steinsson (2018), I assume that the real yield curve rates move one to one with their nominal counterparts so that the new measure is computed simply by summing TIPS to nominal debt $\frac{\sum_{j=0}^{\infty} \frac{j}{12}(q_{t,j} d_{t,j} + \bar{q}_{t,j} \bar{d}_{t,j})}{GDP_t}$. The overall behavior is similar, with the TIPS mattering mildly only in the final period.

The right panel of Figure C.2 presents the same figure for Macaulay duration. The overall pattern and differences across measures are the same as for duration-to-GDP. I would like to highlight only the difference between the baseline and the measure which includes FED holdings. Before QE, the government was holding similar bonds as the general public, so that the two duration measures line up together. However, from 2010 they started to diverge, with the treasury extending maturities, but the FED effectively reducing the supply of long term debt available to the general public. Notice however, that this decline was not sufficient to lower the insurance mechanism of long debt for the treasury, as duration-to-GDP was still increasing in this period. This shows how looking only at Macaulay duration can be misleading, as one would conclude that the insurance mechanism was declining relative to the pre-QE period by looking at Macaulay duration.

Figure C.3 presents the underlying data before being aggregated at the baseline duration-to-GDP measure, nominal marketable public debt promises held by the public excluding t-bills. It shows for each period and maturity, both at monthly intervals, the amount over GDP of the market value of each debt promises over GDP: $\frac{q_{t,j}d_{t,j}}{GDP_t}$. These numbers may look small as they represent the share of GDP owed at each future month. Moreover, debt issuers have tried in recent year to span the yield curve, leading to a high number of relatively small issuances. We can also notice how between the late seventies and the early nineties the Treasury was not issuing long debt at all.

Figure C.4 presents the share of public debt at different maturities. The underlying data is as in the baseline, nominal marketable public debt promises held by the public excluding t-bills. It presents public debt promises shares in four bins with thresholds at 5, 10, and 20 years. We can see how the largest share of public debt is debt below 5 years, even if we exclude t-bills. The reason is that the Treasury issues short dated debt and the fact that bonds carry coupons creases a number of cash flows early on even for long dated debt.

UK Data Description. The right panel of Figure C.5 presents duration-to-GDP and Macaulay duration for the UK and this figure is the counterpart to Figure 1 for the US. It shows nominal marketable debt. As for the US it excludes inflation-linked bonds, t-bills, and non-marketable debt. The key difference with the US data is that for the UK we have only outstanding debt and cannot net out Bank of England or public sector holdings of Gilts. This is not an issue for the main estimation period which ends in December 2007, as substantial holdings of Gilts by the Bank of England started with QE, that is from 2009. The only caveat comes from inspecting Figure C.5, the numbers from 2009 onward are likely to be overestimated. To give an order of magnitude, in September 2019 the Bank of England held 23% of all gilts, so we might have to lower the numbers from the QE period by roughly a fourth (assuming that the Bank of England held the market in terms of maturity split). By inspecting Macaulay duration, we can see how duration at face value from the right panel of Figure C.6, has been declining constantly from 1969 up to the early nineties. However, at market value, we can see much smaller swings in this period due to the high interest rates. Macaulay duration declined substantially in the mid seventies with the increase in interest rates and it has been increasing since despite the decline in face value. From the mid nineties onward we can see an increase in both Macaulay duration at market value and at face value, with the increase at market value being steeper due to the declining interest rates. With respect to duration-to-GDP, we can see in the pre-2009 period the behavior going in phases. Duration to GDP was relatively high before 1974, between the late seventies and the late eighties, and then in the late nineties and late aughts. On the other hand, it was low in the mid seventies, in and the early nineties. From the mid nineties there was an upward trend, with lower values relative to trend in the mid aughts. This behavior is quite distinct from the US one, where phases of low and high values happened at lower frequency and different times. The fact that the empirical results are present for both countries with these different time series behaviors is reassuring and give weights to the idea that the maturity of public debt matters for the transmission of monetary policy.

Figures C.5, C.6, C.7, and C.8 presents the same cuts of the data for the UK as for the US. Overall a few points stand out, first of all inflation linked debt has a bigger role for the UK than for the US to compute duration-to-GDP, the reason is that this type of debt tends to have a longer dated life and it has been used extensively by the UK debt management office. We can also see from the baseline Macaulay duration and the shares of debt that the UK issues relatively longer debt than the US on average. This is also true across other countries, with the UK having longer debt than most European countries.

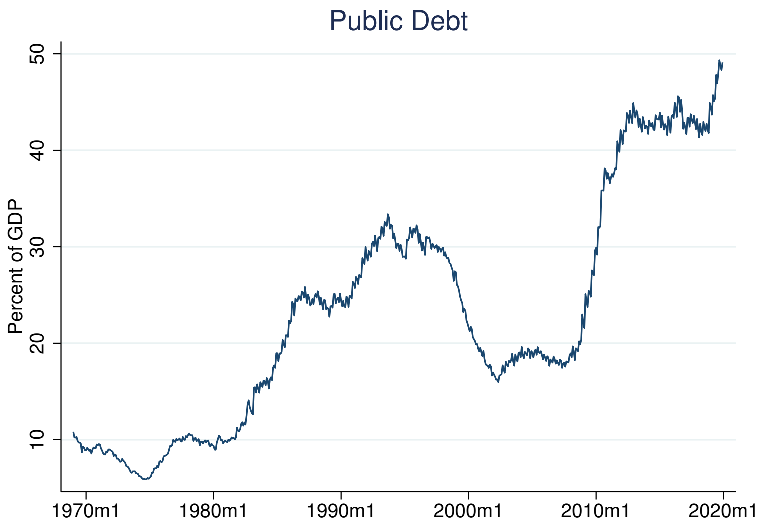
C.2 Monetary Policy Shock Data

The narrative measure of monetary policy shock are in the spirit of Romer and Romer (2004). For the US, the series is the updated Romer and Romer (2004) measure by Yang and Wieland (2015) that spans the period from January 1969 to December 2007. The UK series is the one produced by Cloyne and Hürtgen (2016) from January 1975 to December 2007. These series represent the main constraint in term of sample which I can use in the regressions. The high frequency measure comes from the Proxy-VAR ran by Gertler and Karadi (2015), there, the structural shock is present from July 1980 to June 2012.

C.3 Macroeconomic Data

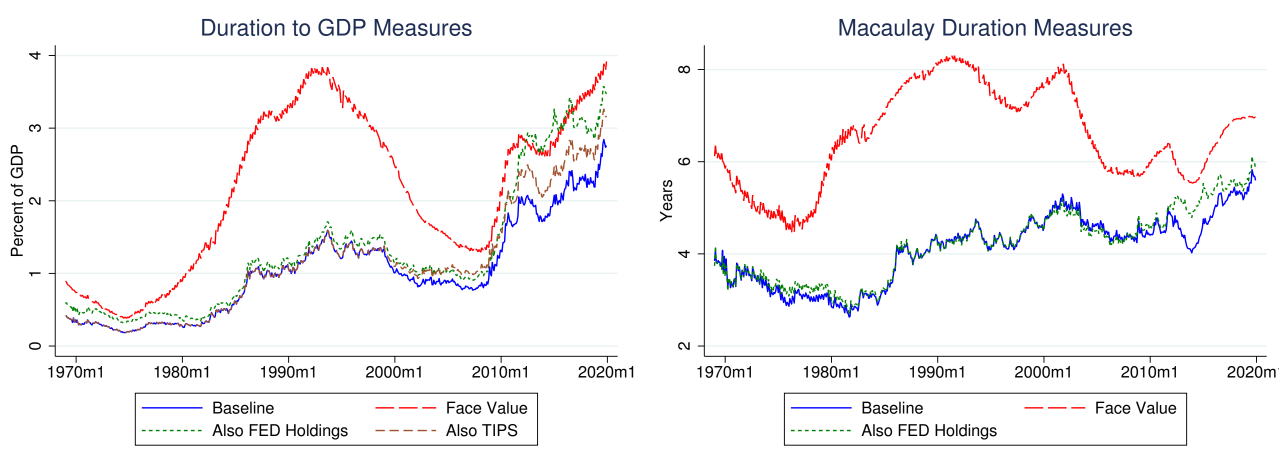
The macroeconomic series used in the various regressions are specified in Tables C.1 and C.2.

Figure C.1: Time series of nominal marketable public debt over GDP held by the public for the US



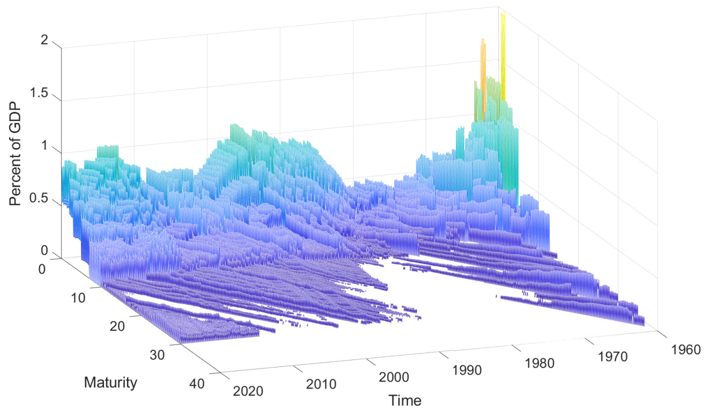
Notes: The figure shows the time series for public debt over GDP for the US. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2019m12 with US data.

Figure C.2: Time series of public debt duration-to-GDP and Macaulay duration for the US with different measures



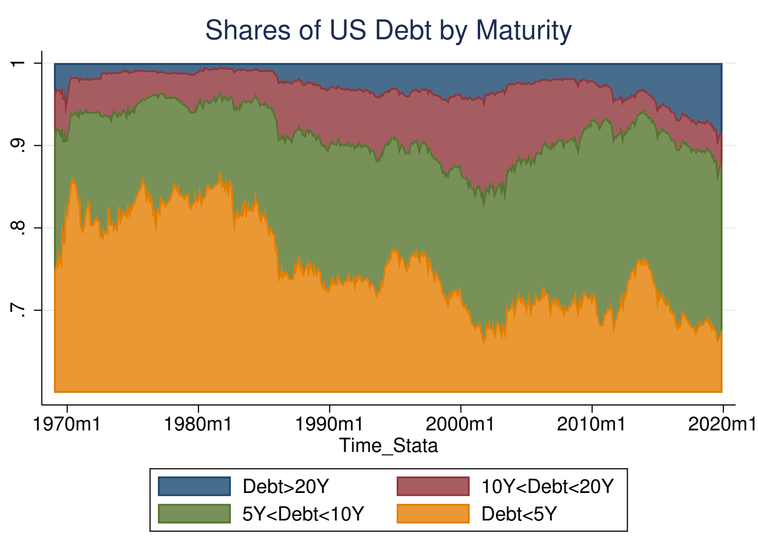
Notes: The figures shows the time series for public debt duration-to-GDP (on the left) and Macaulay duration (on the right) for the US. For the baseline series, the public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration-to-GDP each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). To construct Macaulay duration each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by their market value (the sum without multiplying by maturity). For the face value series each promise is not discounted at market value with yield curve data but is multiplied by one. For the also FED holdings series, I use the overall amount outstanding per each bond and do not net out FED or intra-agencies holdings. For the Also TIPS series, I sum the TIPS as well, this implies that we assume a one to one correlation between nominal yield curve rates and real yield curve rates. The sample goes from 1969m1 to 2019m12 with US data.

Figure C.3: Public debt promises over GDP at various maturities for the US



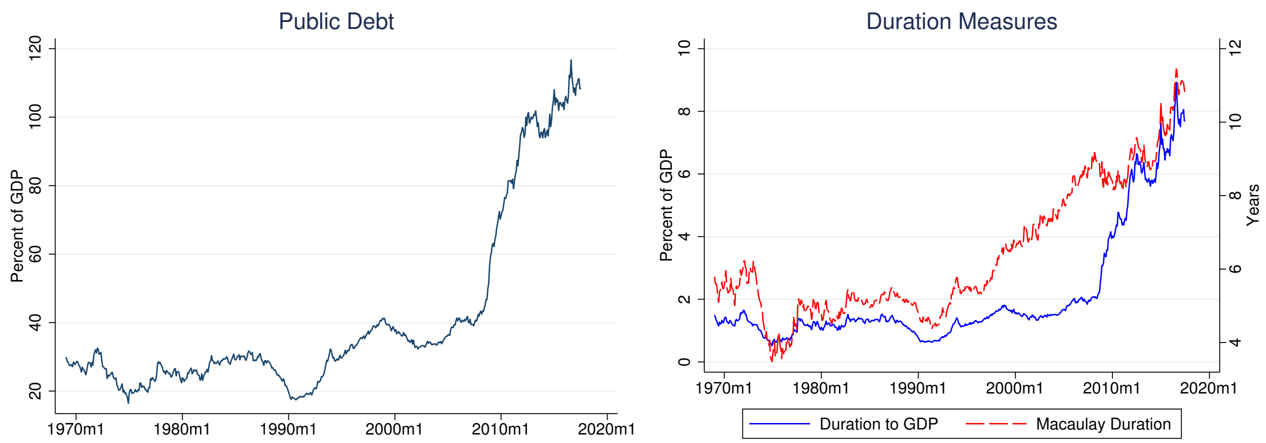
Notes: The figure shows the distribution of public debt promises for the US. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The market value of each promise is deflated by nominal GDP. The time and maturity dimensions are both at monthly frequency. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2019m12 with US data.

Figure C.4: Public debt shares at various maturities for the US



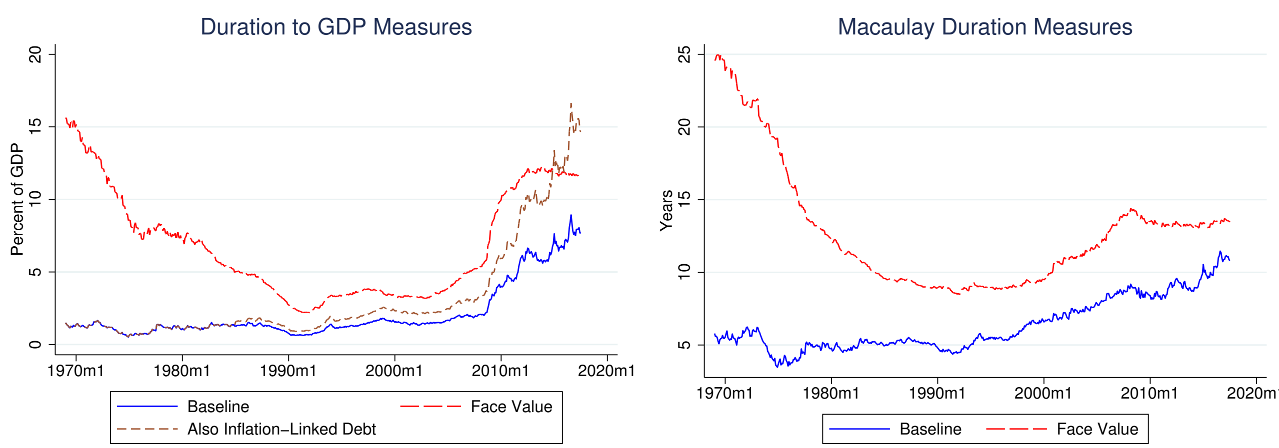
Notes: The figure shows the time series for the shares of debt over GDP for the US within 4 bins: debt below 5 years, debt between 5 and 10 years, debt between 10 and 20 years, and debt above 20 years. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The sum of public debt promises within the bins is divided by the sum of public debt promises across all the bins. The sample goes from 1969m1 to 2019m12 with US data.

Figure C.5: Time series of public debt, Macaulay duration, and duration-to-GDP for the UK



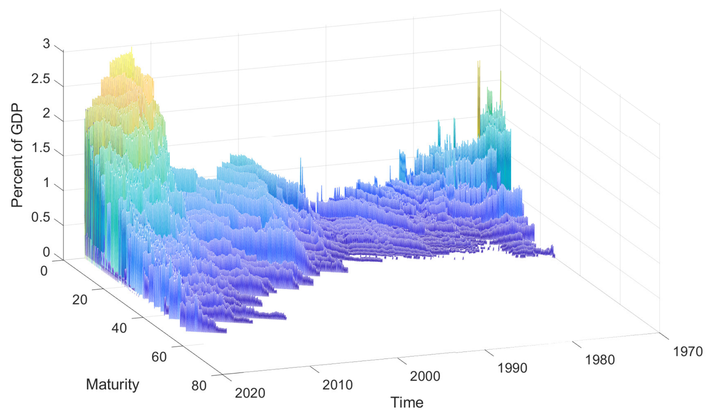
Notes: The figure shows the time series for public debt (in the left panel), Macaulay duration (in red dashed line in the right panel) and duration-to-GDP (in blue solid line in the right panel) for the UK. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration-to-GDP each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2017m7 with UK data.

Figure C.6: Time series of public debt duration-to-GDP and Macaulay Duration for the UK with different measures



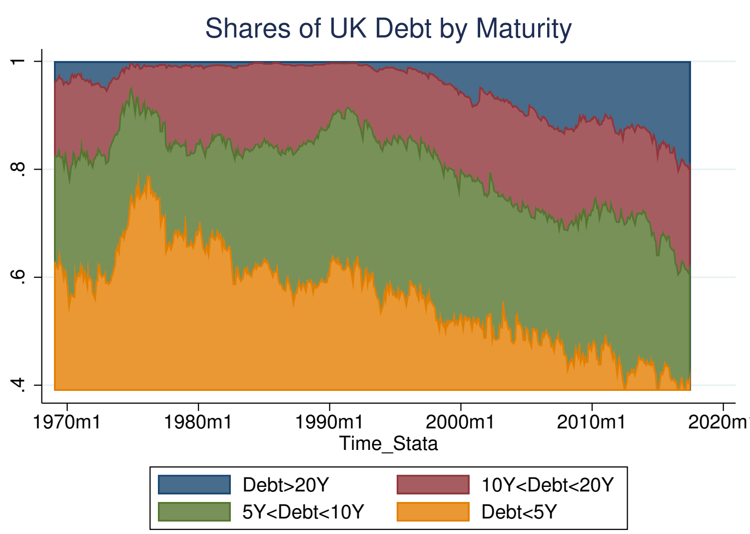
Notes: The figure shows the time series for public debt duration-to-GDP (right panel) and Macaulay duration (left panel) for the UK. For the baseline series, the public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. To construct duration-to-GDP each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). To construct Macaulay duration each public debt discounted promise is multiplied by its maturity in years and then these objects are summed for each period and then divided by their market value (the sum without multiplying by maturity). For the face value series each promise is not discounted at market value with yield curve data but is multiplied by one. For the also inflation linked debt series, I sum the inflation linked debt as well, this implies that we assume a one to one correlation between nominal yield curve rates and real yield curve rates. The sample goes from 1969m1 to 2017m7 with UK data.

Figure C.7: Public debt promises over GDP at various maturities for the UK



Notes: The figure shows the distribution of public debt promises for the UK. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The market value of each promise is deflated by nominal GDP. The time and maturity dimensions are both at monthly frequency. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2017m7 with UK data.

Figure C.8: Public debt shares at various maturities for the UK



Notes: The figure shows the time series for the shares of debt over GDP for the UK within 4 bins: debt below 5 years, debt between 5 and 10 years, debt between 10 and 20 years, and debt above 20 years. The public debt used to construct the measure is nominally fixed rate, marketable bonds outstanding, that is it includes Bank of England holdings. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The sum of public debt promises within the bins is divided by the sum of public debt promises across all the bins. The sample goes from 1969m1 to 2017m7 with UK data.

Table C.1: Macroeconomic US Data

Variable	Transformation	Source	Code
Industrial Production	log times 100	FRED	INDPRO
CPI Price Level	log times 100	FRED	CPIAUCSL
Unemployment Rare	as is	FRED	UNRATE
Effective Federal Funds Rate	as is	FRED	FEDFUNDS
GDP	log times 100	FRED	GDPC1
Commodity Price Index	log times 100	CRB Commodity Price Index, downloaded from Ramey (2016)	LPCOM
Government Consumption Expenditures and Gross Investment	log times 100 less lCPI	FRED	GCE
Government current transfer payments	log times 100 less lCPI	FRED	A084RC1Q027SBEA
Federal Tax Receipts	log times 100 less lCPI	FRED	W006RC1Q027SBEA
Federal Budget Surplus	percent over GDP times 100	FRED	M318501Q027NBEA
Bank Loans	log times 100 less lCPI	FRED	BUSLOANS
AAA Spread over 10 Year Treasury	as is	FRED	AAA10YM
BAA Spread over 10 Year Treasury	as is	FRED	BAA10YM
Nonfinancial Corporate Business, Gross Fixed Investment, Flow.	log times 100 less lCPI	FRED	BOGZ1FA105019005Q
Private Nonresidential Fixed Investment	log times 100 less lCPI	FRED	PNFI
Excess Bond Premium	as is	From Gilchrist and Zakrajšek (2012), downloaded from Gertler and Karadi (2015)	EBP
Commercial Paper Spread	as is	Downloaded from Gertler and Karadi (2015)	CP3M_SPREAD
Mortgage Spread	as is	Downloaded from Gertler and Karadi (2015)	MORTG_SPREAD

Notes: The first column shows the name of the variable. The second column shows which transformation has been applied to be used in the empirical analysis. The third column discusses where the variable was retrieved. Finally, the fourth column shows the code of the variable in the source. For variables from FRED this is the code which can be used to retrieve the variable. For the variables from a previous study the code is the name of the variable in the replication files.

Table C.2: Macroeconomic UK Data

Variable	Transformation	Source	Code
Industrial Production	log times 100	BoE: A Millennium of Macroeconomic Data	Index of Industrial Production
RPIX Inflation	12 months inflation of the retail price index excluding mortgage payments	ONS, downloaded from Cloyne and Hürtgen (2016)	RPIX12m
CPI Price Level	log times 100	BoE: A Millennium of Macroeconomic Data	Spliced monthly Consumer Price index, 1914-2015
Unemployment Rare	as is	BoE: A Millennium of Macroeconomic Data	Monthly unemployment rate
Bank Rate	as is	BoE: A Millennium of Macroeconomic Data	Bank Rate
GDP	log times 100	BoE: A Millennium of Macroeconomic Data	GDP at market prices. Chained volume measure, £mn, 2013 reference year prices
Commodity Price Index	log	IMF, downloaded from Cloyne and Hürtgen (2016)	CommodityPriceIndex (log)

Notes: The first column shows the name of the variable. The second column shows which transformation has been applied to be used in the empirical analysis. The third column discusses where the variable was retrieved. Finally, the fourth column shows the code of the variable in the source. For variables from "BoE: A Millennium of Macroeconomic Data" this is the name of the variable in the excel file. For the variables from a previous study the code is the name of the variable in the replication files.

C.4 Flow of Funds Data

In this section, I discuss how I use the flow of funds data for the US to construct the measures of corporate debt issuance and leverage for the non-financial corporate sector. I follow Greenwood, Hanson and Stein (2010) on corporate debt issuance, who build on Baker, Greenwood and Wurgler (2003). They assume that all short debt is refinanced each period and that the new issuance of long debt is equal to the change in the stock of long debt plus 0.025 times the stock of long debt in the previous quarter, which implies an average maturity of 10 years for long non-financial corporate debt. This measure allows to focus on gross issuance, the sum of short debt and long debt issuance, as this is the relevant metric for the theoretical model. The measure is in log real terms times 100. The data source is Table L.103 Nonfinancial Corporate Business of the flow of funds. Short debt is defined as the sum of "Commercial Paper" (FL103169100.Q), "Depository Institution Loans not elsewhere classified" (FL103168005.Q), and bank loans not elsewhere classified," and "Other loans and advances"[47] (FL103169005.Q). Long debt is defined as the sum of "Municipal Securities[48]" (FL103162000.Q), "Corporate Bonds" (FL103163003.Q), and "Mortgages" (FL103165005.Q). Notice that the flow of funds data changes the codes and definitions across iterations. This description is accurate on the Q4 2019 publication (published on March 2020).

With respect to leverage for the non-financial corporate sector I build a measure relating to the debt leverage, in line with the spirit of the model. Leverage is defined as 100 times "Debt Securities and Loans" (FRED code BCNSDODNS) over the sum of "Total Liabilities" (FRED code: TLBSNNCB) and "Corporate Equities" (FRED code: NCBCEL). The measure is in log terms times 100.

D Sensitivity on Main Empirical Results

D.1 Baseline LP results without interaction term

The main empirical results of this paper highlight conditional effects of monetary policy. As a baseline comparison, it is important to show the underlying average effect results, not conditional on debt maturity. For this reason, this subsection presents the LP and LP-IV regression results for the narrative identification from Ramey (2016) for the US.

Figure D.1 presents the replication of the results of Ramey (2016) for the impact of the monetary policy shock on key macroeconomic variables. The regressions incorporate the re-

[47]These are loans from rest of the world, U.S. government, and non-bank financial institutions.

[48]In the context of nonfinancial corporate businesses these are industrial revenue bonds. They are issued by state and local governments to finance private investment and secured in interest and principal by the industrial user of the funds.

cursiveness assumption and for the US have 2 lags of the log of industrial production, the log of the price level, the unemployment rate, the effective federal funds rate, and the log of the commodity price index. We can see that industrial production and the unemployment rate exhibit a mild expansion puzzle at the first months as it was previously documented in the literature. Industrial production decreases by around one percent at peak and inflation starts decreasing only after 2.5 years and reaches a decline of almost 2 percent after 4 years. The unemployment rate increases by 0.2 percentage points at the peak effect.

The IRFs are remarkably similar when we use a local projection instrumental variable (LP-IV) framework. Figure D.2 displays the IRFs when we do not use directly the monetary policy shock proxy in the regression, but when we use it as an instrument for the structural monetary policy shock. The implementation is to instrument the change in the federal funds rate with the narrative shock measure. The regressions present the same controls and recursiveness assumption. The results could be different if the shock proxy measures the true shock with noise. This does not seem to be a concern in this framework as the IRFs are similar across figures D.1 and D.2. Furthermore, the first stage robust F-statistic is high at 39.04, so there does not seem to be any weak instrument problem. We can interpret these IRFs as what is the impact of a monetary policy shock that raises in the current month the effective federal funds rate by 1%. Industrial production declines by at most 1.6 percent at around one year, the price level declines by 2% after 4 years, and the unemployment rate increases by 0.7 percentage points, again at around one year.

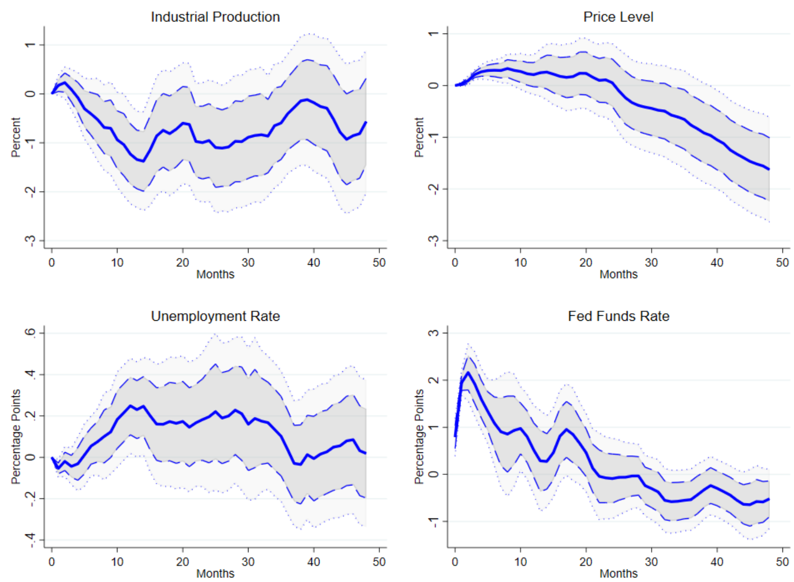
D.2 Quarterly Frequency

In this section, I run the regressions at quarterly frequency. Figure D.3 shows the reduced form regressions for the local projection without any interaction term at quarterly frequency[49]. In these regressions, I added GDP as a dependent variable and as a control with two lags and the recursiveness assumption. Additionally, I add a dummy for each quarter to control for seasonal effects. We can see that industrial production declines at a peak of 1 percent, whereas GDP declines by -0.5 percent. For both variables the peak decline happens after around one year. The price level declines by about 1.5 percent after two years. Finally, the Federal Funds rate increases by more than 1.5 percentage points. The overall results are similar to the monthly ones.

Figure D.4 presents the interaction regression results. GDP declines by 1% when all debt is short term, 1 year following the shock. Similarly, the interaction coefficient is positive and statistically significant: when public debt duration is one standard deviation higher, GDP is 0.6% relatively higher following a contractionary monetary policy shock. The results for

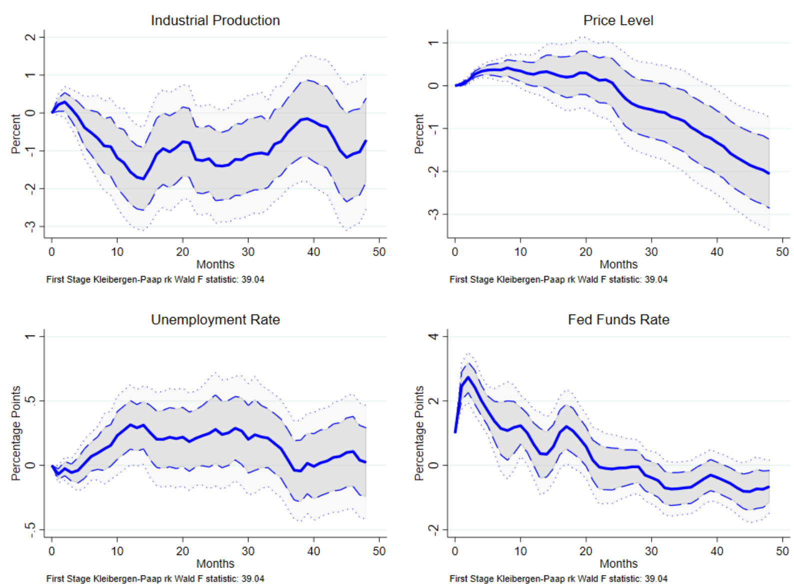
[49]Results with LP-IV are very similar and available upon request.

Figure D.1: Unconditional local projection regressions for the US



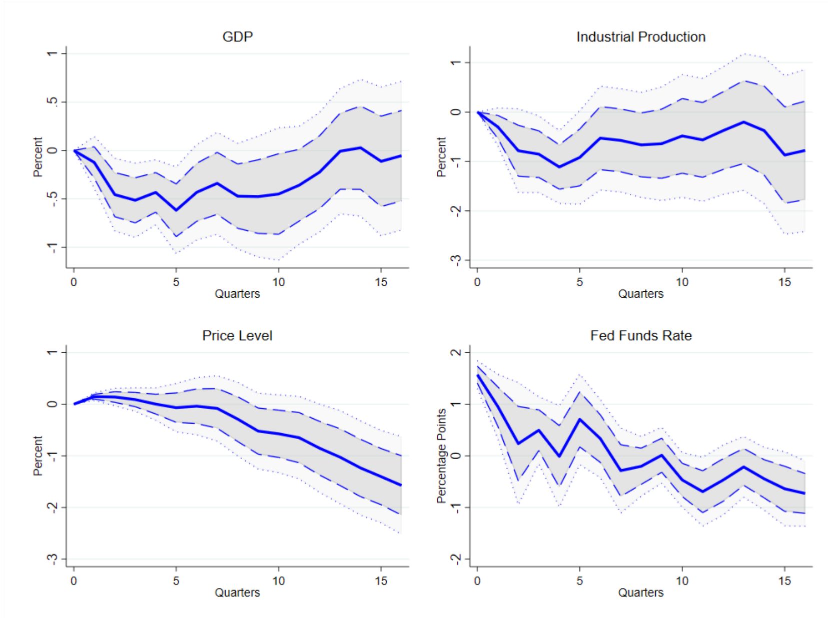
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

Figure D.2: Unconditional local projection instrumental variable regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variable is the change in the Fed funds rate. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

Figure D.3: Unconditional local projection regressions for the US at quarterly frequency



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969q1 to 2007q4 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate.

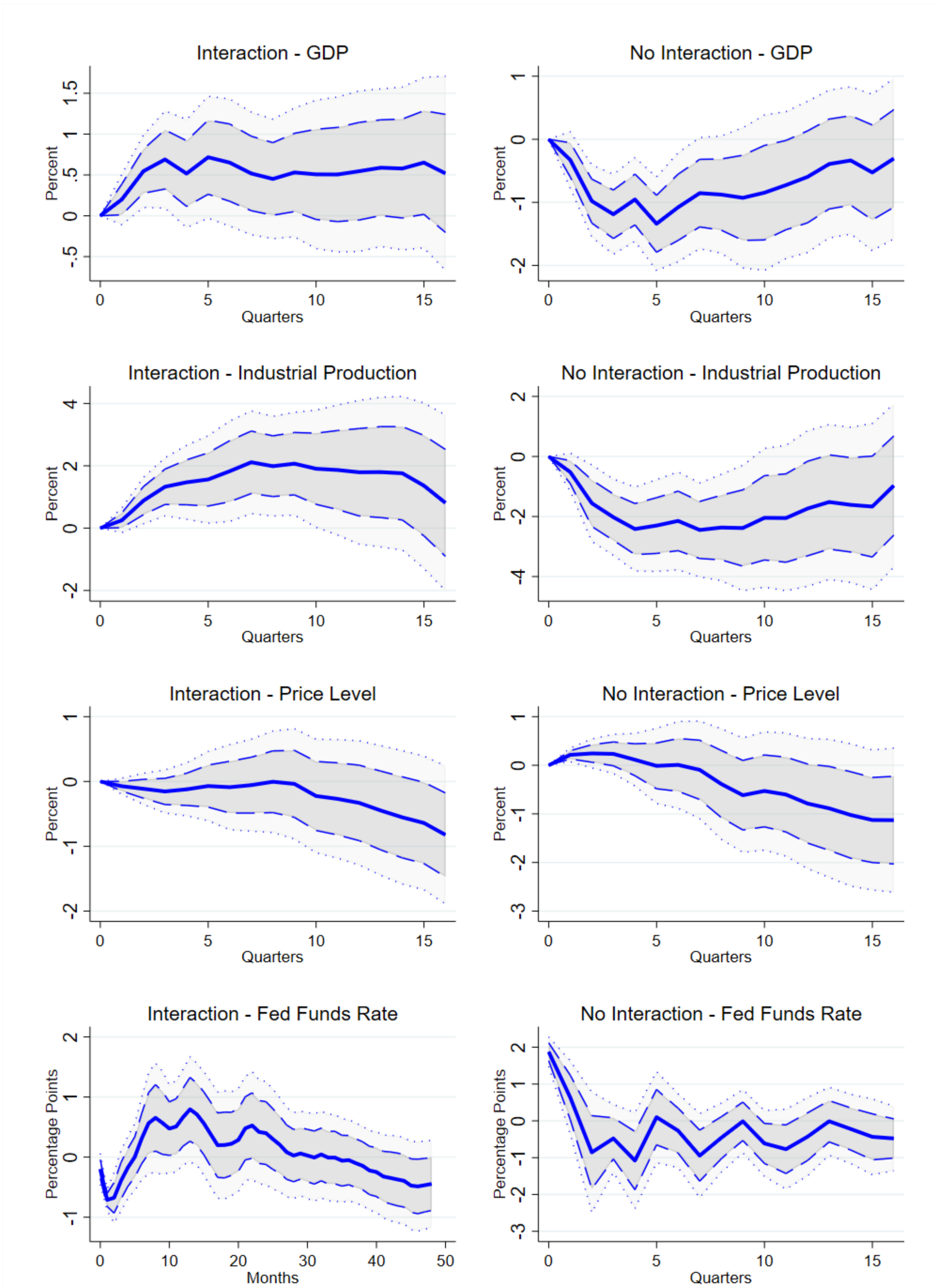
industrial production, for the price level, and for interest rates are virtually identical to the monthly ones.

D.3 Local Projection Instrumental Variables Results

Local projection regressions using the shock measure directly have the benefit of being very transparent and of not imposing any normalization on the effect of the monetary policy shock contemporaneously on interest rates. On the other hand, with an instrumental variable approach we normalize the monetary policy shock to have the impact of increasing interest rates by one percent on impact, this implies in the current framework that a monetary policy shock cannot have a differential effect on interest rates on impact depending on the level of duration-to-GDP. In Figure 2, we could see that interest rates increase mildly less if there is relatively longer duration-to-GDP. For this reason I present the baseline results with reduced form local projections, however, LP-IV have a number of advantages. First of all, if the instrument is measured with noise, inference is valid under LP-IV but might be biased under LP. Moreover, LP-IV provides a test of instrument strength, which is particularly useful for the interaction term I am proposing.

Figure D.5 presents the results for the interaction term coefficients of (3), $\beta_{2,h}$, and Table D.1

Figure D.4: Local projection baseline quarterly interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969q1 to 2007q4 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, GDP, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, GDP, the price level, the commodity price index, the unemployment rate, and the Fed funds rate, one lag of duration-to-GDP, and a dummy per quarter. The first column shows the interaction term of the shock with the Duration-to-GDP, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

presents the first stage regression coefficients of (3): γ_{11}, γ_{12}, γ_{21}, and γ_{22}. The LP-IV presented here follow the same specification as the baseline LP presented in Figure 2: monetary policy shocks are estimated with the updated Romer and Romer method. Each regression incorporates the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. Finally, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP.

From Figure D.5, we can see how the interaction coefficients are very close in magnitude, path and significance to the reduced form coefficients presented in Figure 2. An increase by one standard deviation of duration-to-GDP attenuates the effect of a contractionary monetary policy shock by almost 3% at peak after 2 years. There is no statistically significant differential effect of public debt duration for the effect of monetary policy on the price level. The mediating effect on industrial production is similar on unemployment, whereby at peak the interaction coefficient reaches -0.7 percentage points. The small increase in magnitude in the LP-IV compared to the LP results in the interaction regressions is in line with the difference in magnitude we can see in the linear average regressions.

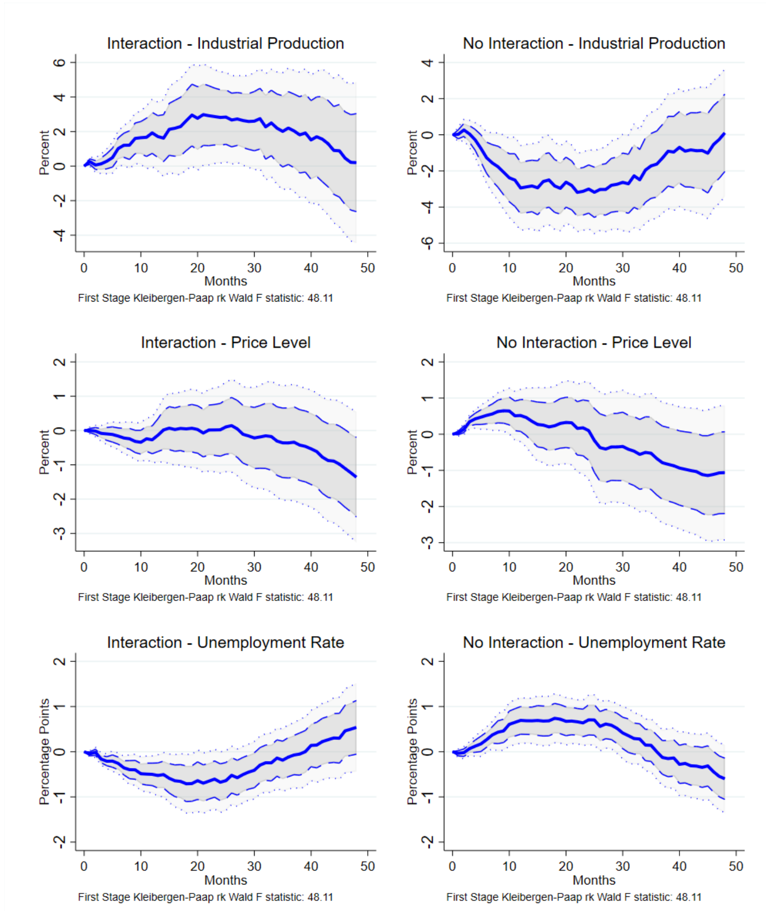
The first stage Kleibergen-Paap robust F-stat is high at 48.02, which is well above the 10 rule of thumb for weak instruments. Notice that this F statistic tests jointly if $Shock_t$ and $Shock_t DurGDP_{t-1}$ are a strong set of instruments for $\Delta i_{t,t-1}$ and $\Delta i_{t,t-1} DurGDP_{t-1}$. This is the relevant statistic for the object of interest, however, it is still informative to examine the first stage more in detail. The first 2 columns of Table D.1 do that, they show γ_{11}, γ_{12}, γ_{21}, and γ_{22} with the same controls as in Figure D.5. In addition, columns (3) and (4) show the same first stage regressions while controlling only for $DurGDP_{t-1}$ as a robustness check. From examining the first two columns we can see how the instrument on its own has a strong, almost one to one, effect on the change in the federal funds rate. Moreover, when duration-to-GDP is higher, the effect of a monetary policy shock on impact is lower[50]. Similarly, the main effect on $\Delta i_{t;t-1} DurGDP_{t-1}$ is due to $Shock_t DurGDP_{t-1}$, with a positive, and statistically significant impact. The results columns (3) and (4) show that the results are robust in excluding the macroeconomic controls.

D.4 Test on Monetary Policy Effects when Duration to GDP is High

From Figures 2 and D.5 we can conclude that monetary policy has stronger contractionary effects when duration-to-GDP is low, and that increasing duration-to-GDP lowers the contractionary monetary policy effects on industrial production. However, as the coefficient estimated on the interaction is high, one might wonder if the effect of an increase in interest rates due to

[50]This mirrors the last row of Figure 2.

Figure D.5: Local projection instrumental variable baseline interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The instrumented variables are the change in the Fed funds rate and the interaction between the Fed funds rate change and the lagged duration-to-GDP. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. Each panel shows the interaction coefficient between the instrumented change in the Fed funds rate and duration-to-GDP.

Table D.1: First Stage Regressions

VARIABLES	(1) $\Delta i_{t;t-1}$	(2) $\Delta i_{t;t-1} DurGDP_{t-1}$	(3) $\Delta i_{t;t-1}$	(4) $\Delta i_{t;t-1} DurGDP_{t-1}$
$Shock_t$	0.963***	0.203	1.074***	0.349*
	(0.219)	(0.152)	(0.299)	(0.202)
$Shock_t DurGDP_{t-1}$	-0.198*	0.455***	-0.172	0.454***
	(0.101)	(0.101)	(0.132)	(0.121)
Observations	467	467	467	467
Controls	Recursive	Recursive	Minimal	Minimal

Notes: Newey-West standard errors in parentheses. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. The depended variables are the instrumented variables in the LP-IV framework; they are the change in the Fed funds rate and the interaction between the Fed funds rate change and the lagged duration-to-GDP. The first two columns show the first stage regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, these two regressions include the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The second two sets of regressions show the same first stage regressions with minimal controls, that is, controlling only for one lag of duration-to-GDP.

a monetary policy shock turns positive on economic activity when duration-to-GDP is high. In this section, I present evidence that this is not the case. Specifically, when duration-to-GDP is high the effect of an increase in interest rates turns insignificant on industrial production.

The hypothesis tested is whether at each horizon h the effect of a monetary policy shock is different from zero when duration-to-GDP is one standard deviation above its historical mean. I conduct the test both with the reduced form specification in (2) and the LP-IV in (3). The empirical specification is the same as in Figures 2 and D.5 for LP and LP-IV respectively, with Newey and West (1987) standard errors, the recursiveness assumption, and two lags of the macroeconomic controls. If we re-run the specification with duration-to-GDP standardized the test is simply: $H_0 : \beta^{std}_{1,h} + \beta^{std}_{2,h} = 0$ for each horizon $h = 0, \cdots, H$.

The results are economically the same across the LP and LP-IV specifications. The p-value associated with the test is smaller than 0.05 only for the first month following the shock, at $h = 1$. This is due to the activity puzzle that we can see also on the linear average regressions in D.1 and D.2. For each following horizon ($h = 2, \cdots, 48$) we have p-values all above 0.10, indicating that we cannot reject the hypothesis that monetary policy effects on economic activity are not statistically different from zero at the 90% confidence level.

Overall, we can conclude that monetary policy has relatively lower contractionary effects on industrial production when duration-to-GDP is higher, and for high levels of duration-to-GDP the effects turn insignificant.

D.5 Results with Macaulay Duration

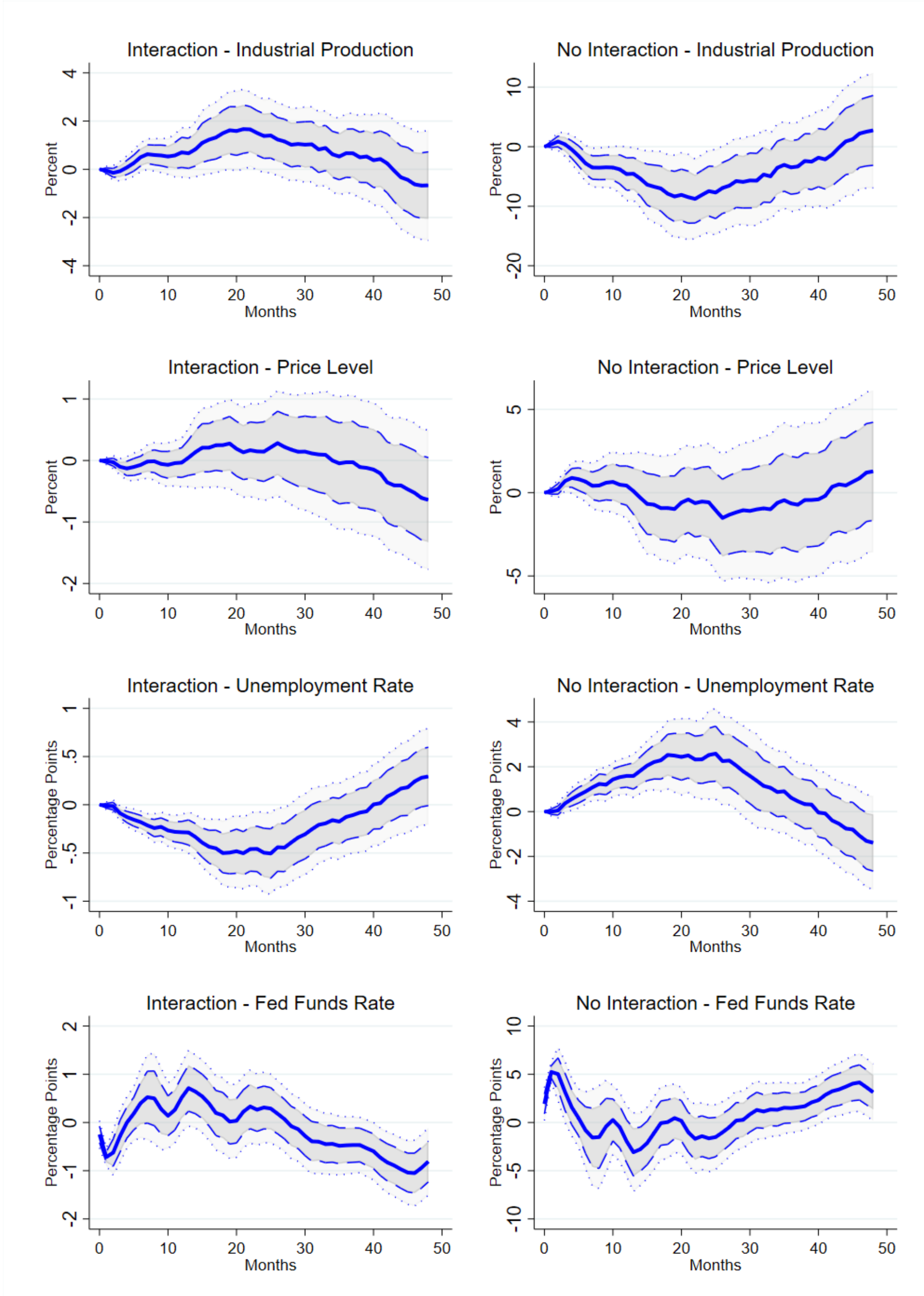
Figure D.6 shows a robustness check where the standard Macaulay duration replaces the Macaulay duration-to-GDP. This duration is measured in years and the interaction coefficients still show the Macaulay duration divided by its own standard deviation. Overall results point in the same direction as the those with the new Macaulay duration-to-GDP but are generally less precisely estimated. This is not surprising, as what matters for the insurance mechanism of the maturity structure is the overall amount of insurance: the amount of insurance over GDP, and not per unit of debt. The coefficients on the first column represent how much more (or less) is monetary policy effective on the left hand side variable when public debt has a one standard deviation longer duration. The no interaction term in the second column refers to the impact of monetary policy on a left hand side variable when the government has a zero years maturity of public debt. The closest empirical counterpart would be to have all public debt that needs to be refinanced overnight. On the first row, we can see the impact on industrial production. At peak having a one standard deviation longer debt Macaulay duration implies having a monetary policy which is more than 2% less effective on output. This coefficient is less precisely estimated than the coefficient on Figure D.1. If all government debt were overnight the impact on industrial production would be massive at almost -10% at peak, but this coefficient is badly estimated. We can find similar results for all remaining variables, the direction of each IRF is the same as in Figure D.1, but the estimates are much less precise.

D.6 Results with Public Debt

Duration to GDP is the measure that correctly captures the insurance mechanism of fixed rate long maturity debt from the perspective of the fiscal authority. The reason is that Macaulay duration itself captures this insurance mechanism per unit of debt, that is, by how much the market value of one unit of public debt would increase following a one percent decline in interest rates. By scaling the measure to the overall amount of public debt to GDP in the economy we can find the relevant metric for the fiscal authority, which cares about the insurance on interest payments over GDP. An interesting question that arises is whether it is possible to separately identify the roles of Macaulay duration and debt to GDP. Unfortunately, this is quite challenging with the current strategy. The reason is that we would need to use three highly collinear variables in the local projection regressions: the monetary policy shock, the interaction of the monetary policy shock with Macaulay duration, and the interaction of the monetary policy shock with public debt to GDP.

Figure D.8 presents out of completeness the results of this exercise. We can see that IRFs show large swings generally associated with multicollinearity. The first column shows the interaction term of the monetary policy shock with Macaulay duration, the second column the interaction term the monetary policy shock with public debt to GDP, and the third column the coefficients associated with the monetary policy shock alone. The first row shows industrial production as a outcome variable, the second the price level, the third unemployment, and

Figure D.6: local projection regressions with Macaulay duration for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the interaction term of the shock with the Macaulay Duration, the second column shows the shock term not interacted. Each row shows a different LHS variable.

the last the Fed funds rate. We cannot see much for industrial production, as there are large swings at the end of the sample. Moreover, the price level regressions are not significant across the board, and the Fed funds rate regressions swing around due to multicollinearity. The only interesting results can be seen in the initial periods (up to the third year) of the unemployment regressions. There, we can see how higher duration of public debt leads to a lower effect of monetary policy on unemployment, there does not seem to be an effect of debt to GDP, and the no-interaction regression points to a contractionary effect of monetary policy on unemployment when there is debt tends to zero and any such debt is overnight. The unemployment results point to the mediating role of maturity once we control for debt levels. However, the results are quite unstable due to multicollinearity so should only be taken as suggestive.

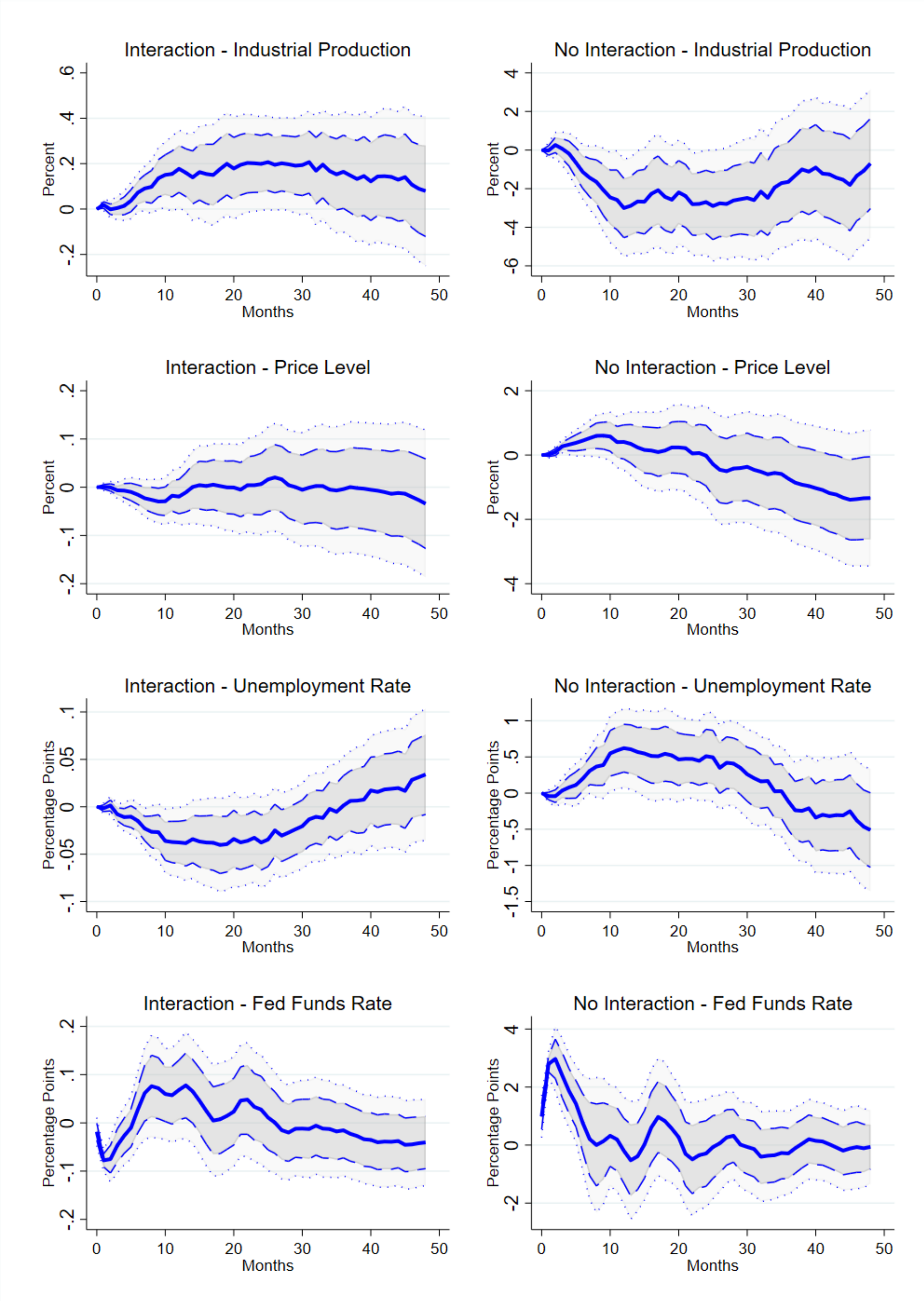
As an additional robustness check, I present results also for public debt over GDP on its own. Public debt is constructed from the same data as the duration measures for consistency, I only use marketable nominally-fixed rate bonds held by the general public. As public debt over GDP is correlated with the Macaulay duration, it is not possible to identify separately the effect of public debt. However, a hint that what matters is the insurance of public debt measured as duration-to-GDP is that we can identify more precisely the coefficients of the baseline regressions with duration-to-GDP in Figure 2 rather then those with public debt in Figure D.7. Overall, we can see all IRFs pointing to the same direction, but the interaction coefficients for industrial production and unemployment (both indicating less contractionary monetary policy with more debt) are less precisely estimated in Figure D.7.

D.7 Results with Duration to GDP Computed from Alternative Debt Definition

In the baseline specifications, I presented the results of duration-to-GDP for a subset of public debt: nominally fixed rate marketable bonds at market value held by the general public. The reason for this choice is a mix of data availability and that this measure is the most suited for the problem at study.

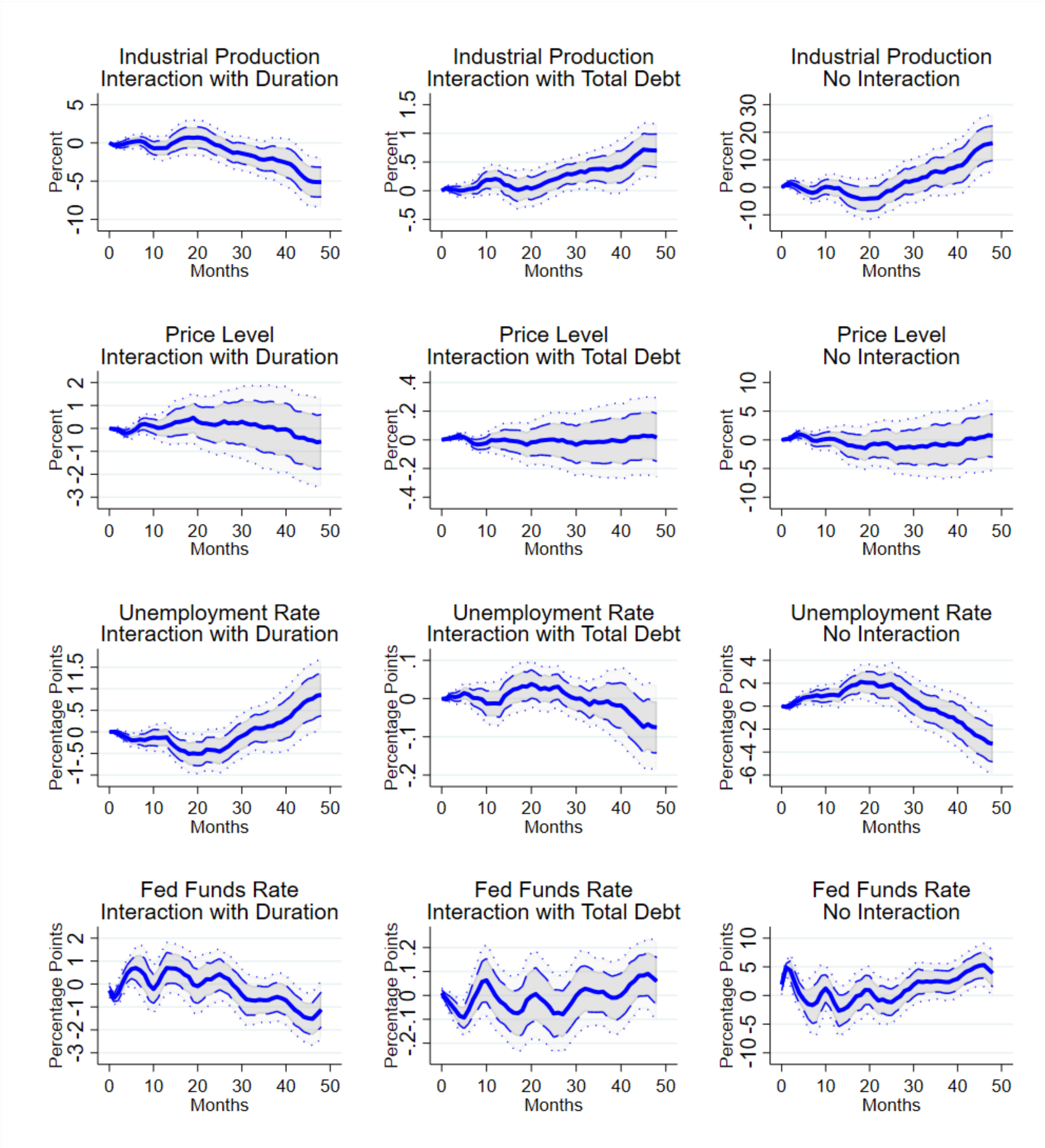
I use marketable bonds as there is data that allows to compute for each of these bond the principal and coupon payments. The exclusion of non marketable debt should not be a concern as, in the period I consider, most of US and UK public debt was marketable, the last periods in which non marketable debt paid an important role as during World Wars and during the Korean War for the US. Moreover, I exclude treasury bills which are not present in the data I use. This is not very problematic as my main statistic of duration over GDP is only mildly affected by securities with very short maturity as treasury bills. Existing treasury bills prices are not strongly affected by changes in interest rates. We can see this mathematically in (1), as the securities with short maturity j are weighted by a low value j. Furthermore, I divide

Figure D.7: local projection regressions with Public Debt for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of public debt to GDP. The first column shows the interaction term of the shock with the debt to GDP computed from nominally fixed rate marketable public bonds held by the general public, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure D.8: local projection regressions with Public Debt and Macaulay Duration for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of Macaulay duration and public debt to GDP. The first column shows the interaction term of the shock with the Macaulay Duration, the second column shows interaction term with debt to GDP computed from nominally fixed rate marketable public bonds held by the general public, and the third column shows the shock term not interacted. Each row shows a different LHS variable.

(1) by GDP, which is not affected by treasury bills[51].

Duration to GDP is computed with the market value of public debt as this measures how much would public debt to GDP increase with a one percent decline in interest rates across the yield curve. Furthermore, I use the lagged value of duration-to-GDP and a plausibly exogenous monetary policy shock, which under the identification assumption, cannot be forecasted with prior information. Consequently, the use of market prices should not weaken identification. In a different context, Greenwood and Vayanos (2014) propose to use a similar metric with face value debt promises:

$$DurGDPFaceValue_t = \frac{\sum_{j=0}^{\infty} j d_{t,j}}{GDP_t}$$

This metric does not have a direct interpretation[52] as the baseline one but has the benefit being more stable in time. For this reason, Figure D.9 presents the same regression results as 2 with the alternative metric of duration-to-GDP at face value. The results point to the same direction as in the baseline: a contractionary monetary shock reduces output less the higher duration-to-GDP at face value. Similarly, a contractionary monetary attenuated the increases in unemployment and does not have an effect on the transmission to the price level. The difference with the regressions with duration-to-GDP at market value is that the coefficients on the interactions on industrial production and unemployment are less precisely estimated.

The baseline statistic of duration-to-GDP uses only nominally-fixed rate treasury bonds and excludes inflation-linked TIPS. The reason is that we need an additional assumption to interpret the results with TIPS debt included. Specifically, duration-to-GDP with nominal debt can be interpreted as how much nominal public debt over GDP increases following one percent decrease in interest rates across the yield curve. In order to add TIPS we need to make an additional assumption on how much the real yield curve (on TIPS) decreases across maturities following a one percent decrease in interest rates across the nominal yield curve. In the exercise that follows I assume a one to one increase, in line with the findings of Nakamura and Steinsson (2018) following a monetary policy shock. The resulting formula for duration-to-GDP with TIPS included is:

$$DurGDPwithTIPS_t = \frac{\sum_{j=0}^{\infty} \left[j p_{t,j} d_{t,j} + j \bar{p}_{t,j} \bar{b}_{t,j} \right]}{GDP_t}$$

[51] Notice that, Macaulay duration suffers more from the exclusion of treasury bills. The reason is that, with Macaulay duration, one needs to divide by the market value of public debt, which is affected by the inclusion of treasury bills.

[52] Specifically, it gives too much weight to long debt, for a given interest rate the price of long debt much lower than short debt even for moderately positive levels of interest rates.

Notice that, this measure is quite close to the baseline one as the US treasury issues mainly nominal bonds and the issuance of TIPS started only in the last part of the sample, in 1999. Figure D.10 presents the results of this exercise. As expected, the results in sign, magnitude, and significance of all IRFs mirror closely the ones in Figure 2. Monetary policy is less effective on industrial production and unemployment when there is more long term debt but the effect does not go through the price level.

Public debt held by the government sector (e.g. social security or FED) should not matter to explain the results. The reason is that an increase in valuation of debt is a negative news for the treasury as they could have borrowed at a cheaper rate if debt was all overnight, but it is a positive news for bond holders. For any bond held by a government owned entity, the net effect is zero, the loss of one branch is the gain of the other. This is why I exclude the debt held by government entities and focus only on debt held by the general public from my baseline metric. However, one might argue that frictions within the government sector do not allow this consolidation as the gains or losses from government entities are not transmitted to the treasury. To assuage risks associated to this Figure D.11 displays the IRFs where duration-to-GDP is constructed from all outstanding nominal fixed rate marketable bonds. Again, results are remarkably close to Figure 2 in sign, magnitude, and significance across all IRFs. This is not surprising as, prior to the QE era duration-to-GDP of outstanding debt tracked quite closely duration-to-GDP of debt held by the general public. In the post QE era the FED started to intervene heavily in specific market segments, thereby lowering duration-to-GDP for debt held by the general public relatively to duration-to-GDP for debt held only by the general public.

D.8 Results with the Share of Long Debt over GDP

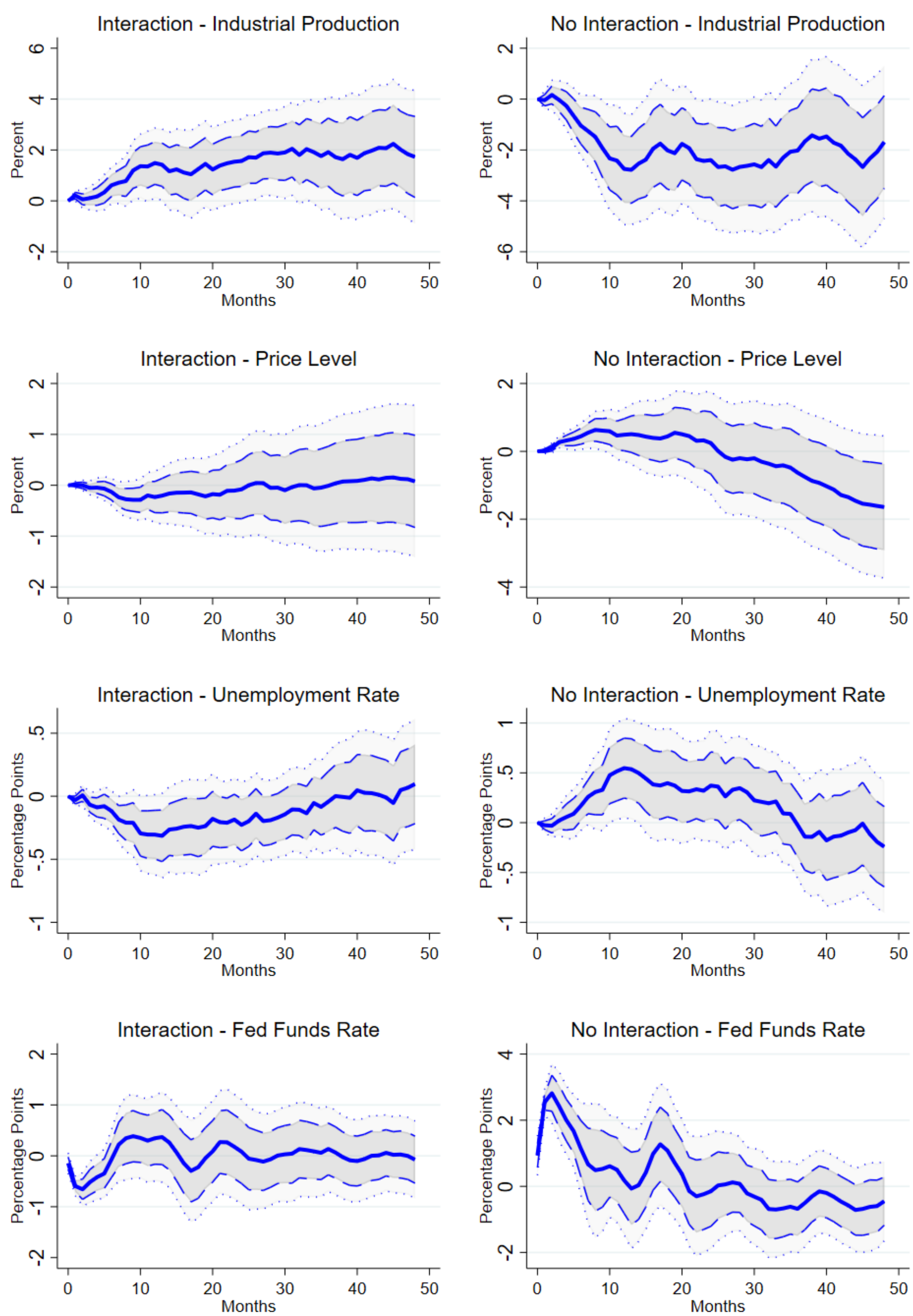
In this section I propose an alternative measure to gauge the size of insurance provided by long debt: the share of debt promises above a threshold over GDP. I construct this measure in each period by summing over all debt promises discounted at the yield curve rate above a threshold, and then dividing this by current nominal GDP. For a threshold ι we can define the measure on monthly data as:

$$LongDebt_{t,\iota} = \frac{\sum_{j=0}^{\infty} \mathbf{1}_{\iota,j} q_{t,j} d_{t,j}}{GDP_t}$$

$$\mathbf{1}_{\iota,j} = \begin{cases} 1 & \text{if } j \geq \iota/12 \\ 0 & \text{otherwise} \end{cases}$$

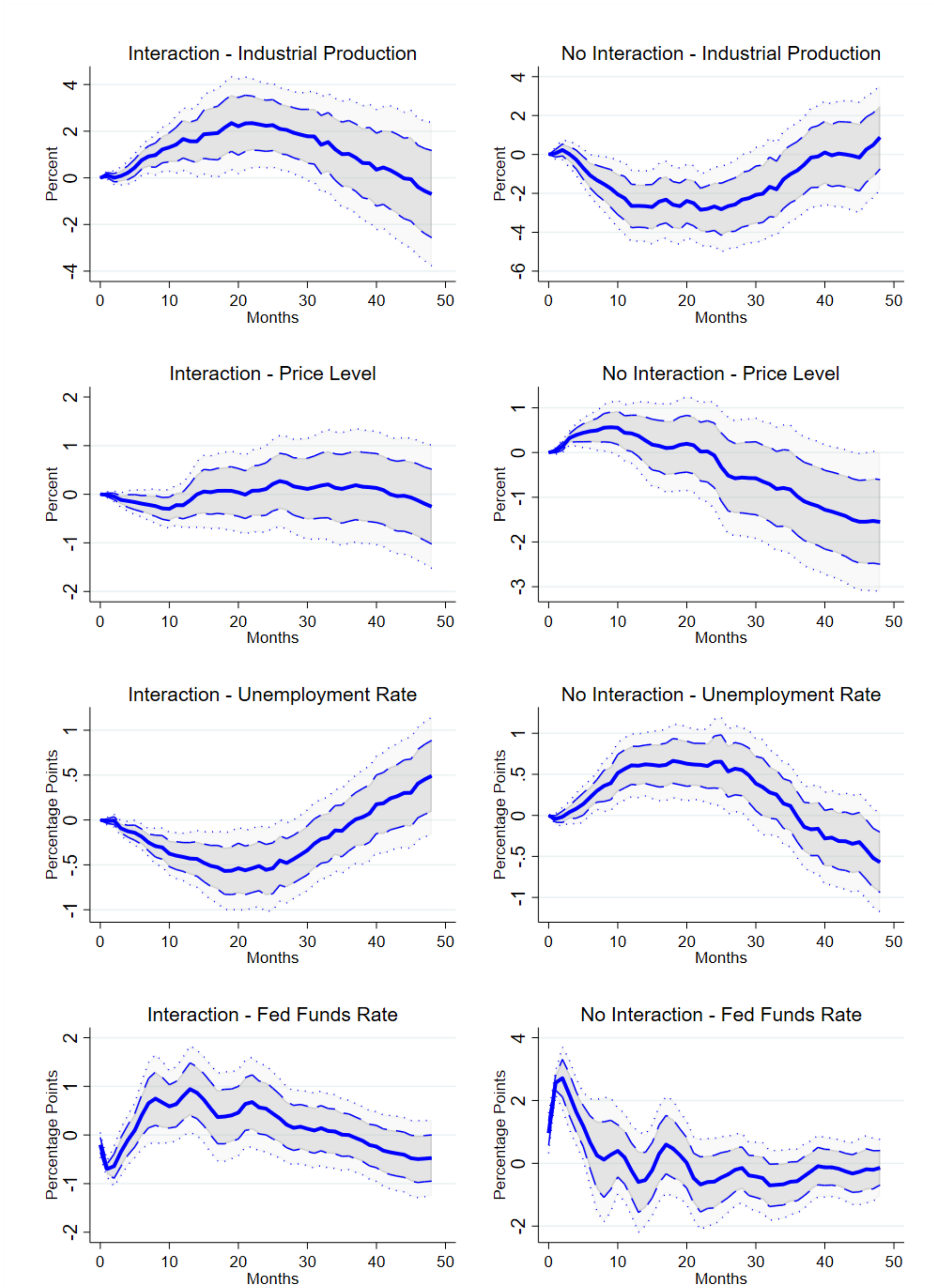
This is similar to duration over GDP with the difference that the indicator function is substituted with $j/12$ in the duration-to-GDP measures. Both measures give more weight to long debt than they do to short debt, with the indicator function giving only zeros and ones. This

Figure D.9: Local projection regressions with duration-to-GDP constructed from face value debt



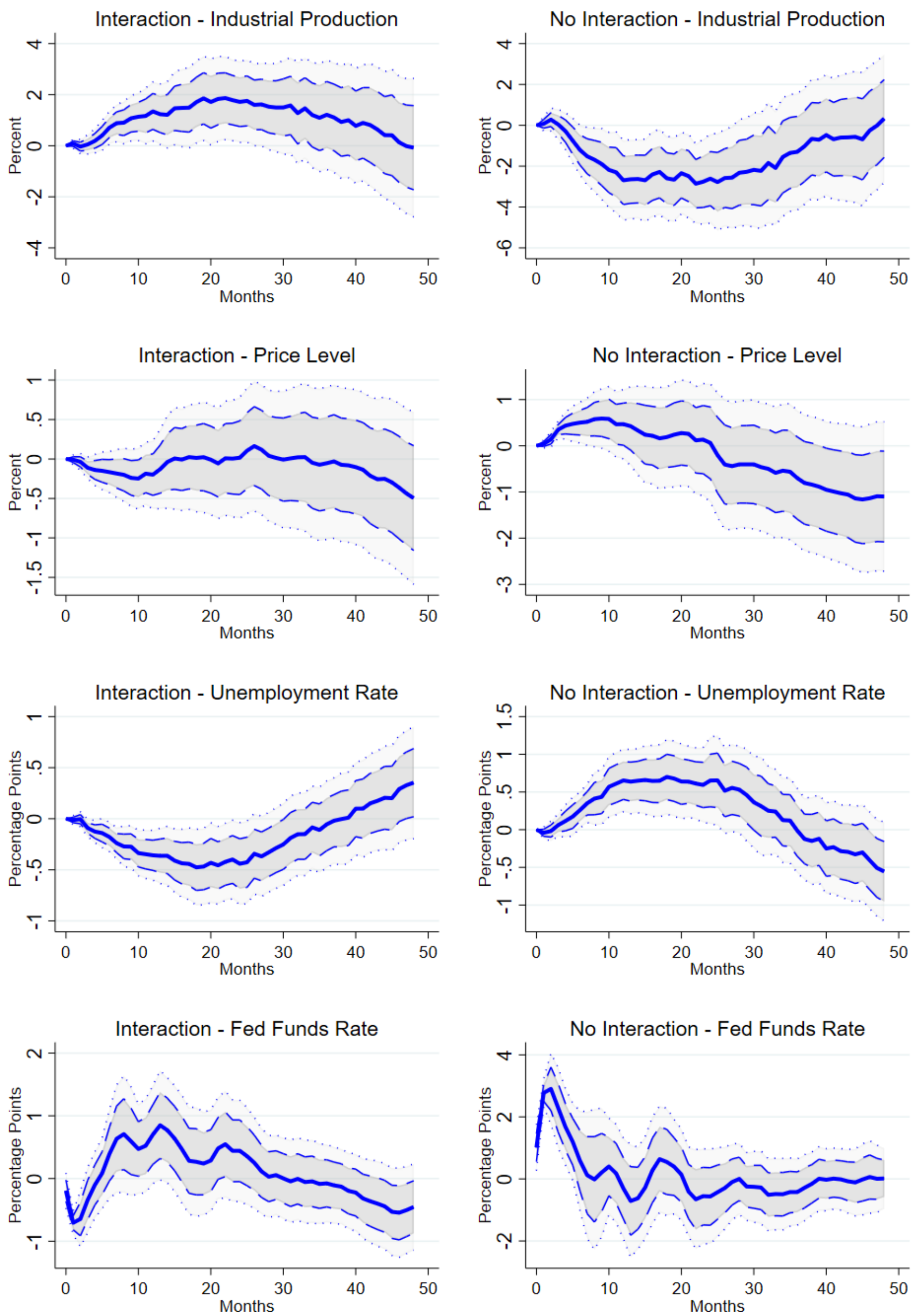
Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the interaction term of the shock with the duration over GDP constructed with face value debt, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure D.10: Local projection regressions with duration-to-GDP constructed from both nominal treasury bonds and inflation linked TIPS



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the interaction term of the shock with the duration over GDP constructed with both nominally fixed rate bonds and TIPS, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure D.11: Local projection regressions with duration-to-GDP constructed from debt held both by the general public and the government sector



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the interaction term of the shock with the duration over GDP constructed with debt held both by the general public and the government sector, the second column shows the shock term not interacted. Each row shows a different LHS variable.

new measure has a few advantages and disadvantages. The main advantage is that it is not sensitive to mismeasurement of short debt. This is relevant in the current context as treasury bills are not present in the data I use. The disadvantages of this measure are that the thresholds are arbitrary and it does not have a direct interpretation as with duration-to-GDP. Duration to GDP measures the increase in market value of public debt to GDP following a one percent decrease in interest rates across the yield curve. Equivalently, it measures what is the present discounted value of interest rate costs over GDP relative increase compared to overnight debt following the same rate change. Long debt above a threshold ι over GDP measures how much debt does not need to be refinanced, or is insured, in the next ι years.

Figure D.12 presents the time series for this measure for two thresholds: 5 and 10 years. We can see that the time series properties are similar to Figure 1. In the first part of the sample the US had very little long debt outstanding, with a large increase in the mid eighties, followed by a slower increase up to the mid nineties. In the last part of this sample there was a gentle decline up to the financial crisis.

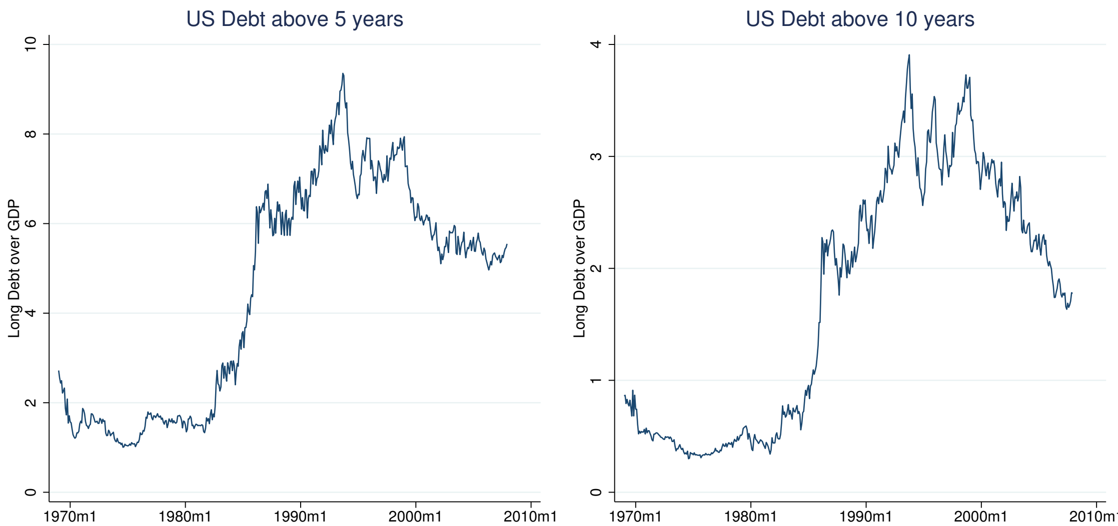
Figure D.13 report the same specification as Figure 2 with the new interaction terms, with a 5 years threshold. The results of this exercise are strikingly similar to the baseline. Monetary policy is less effective on output the higher the amount of long debt in the economy. Being in an economy with one percent more long debt above 5 years over GDP lowers the impact of monetary policy by one percent at its peak. When there is not debt above 5 years monetary policy has stronger effects on industrial production, up to almost 3%. As in the baseline, unemployment behaves as a mirror to industrial production and the amount of long debt does not seem to influence the transmission of monetary policy on inflation. Interest rates increase mildly less than under long debt, especially in months 2 to 4. Results are not sensitive to the threshold, if one uses 10 years they are very similar, these are available upon request.

Overall, measuring the insurance mechanism provided by long fixed rate debt with a more immediate measure as long debt over GDP yields similar results as measuring it with duration over GDP. Longer maturity public debt lowers the effect of monetary policy on output but does not affect its transmission to inflation.

D.9 Results with Deviation of Duration to GDP from Hypothetical one Period Duration to GDP

Figure D.14 presents the baseline results with the interaction term being the deviation of duration-to-GDP from the theoretical duration of public debt, if all debt was issued as a one quarter debt. This allows to interpret the non-interaction regression directly as in the model, where I compare the actual average duration of public debt to a one period (quarter) debt case. The results are virtually identical to the baseline ones.

Figure D.12: Time series of long debt over GDP for the US

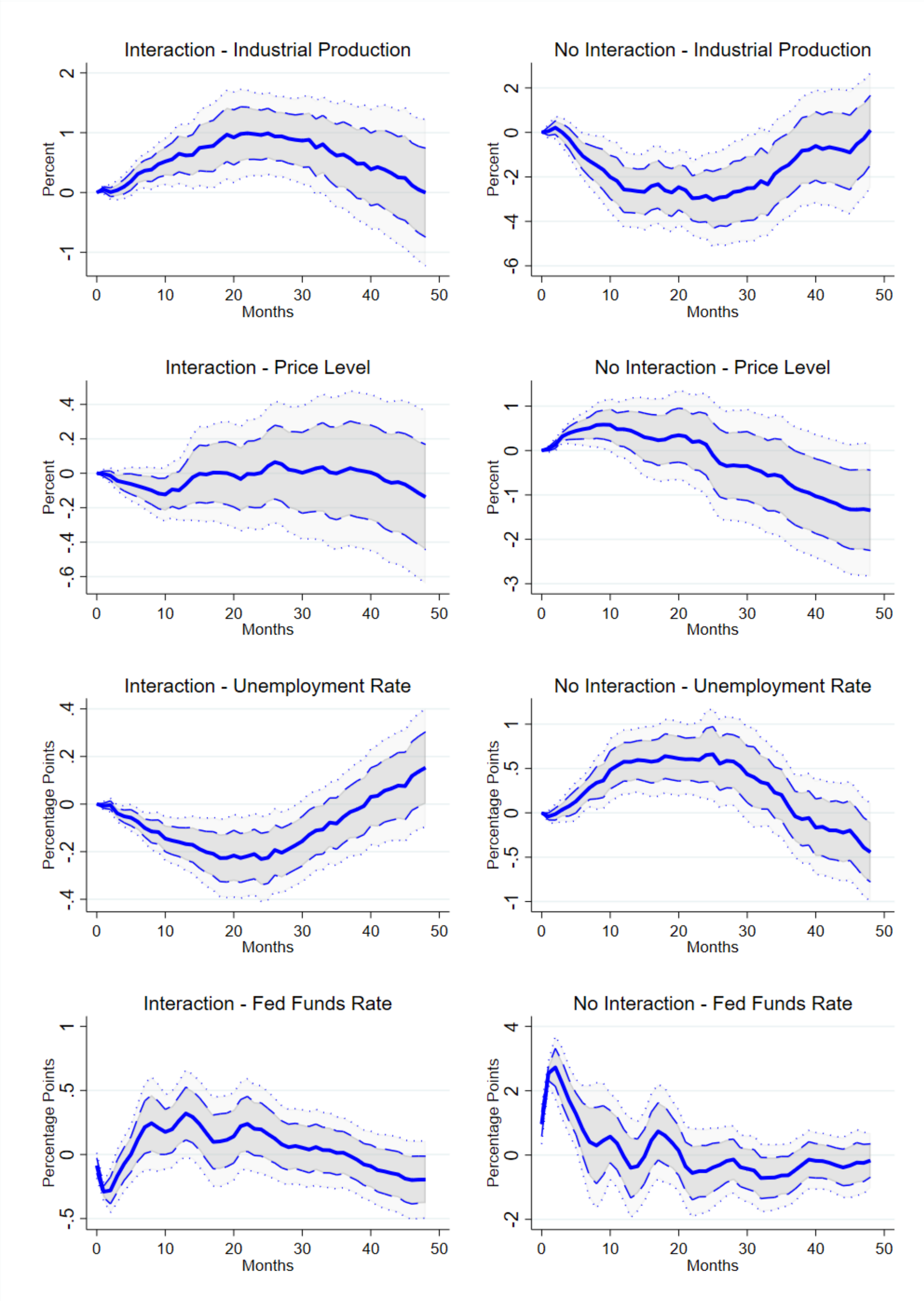


Notes: The figures show the time series for long debt over GDP for the US above two thresholds: 5 and 10 years. The public debt used to construct the measure is nominally fixed rate, marketable bonds held by the public. Each bond is stripped in principal and coupon promises and each promise is discounted at market value with yield curve data. The sum of public debt promises above one of the two thresholds is divided by nominal GDP. GDP is converted from quarterly to monthly values by using the latest value available (e.g. March, April, and May use Q1 values). The sample goes from 1969m1 to 2007m12 with US data.

D.10 Results with Smooth Transition Method

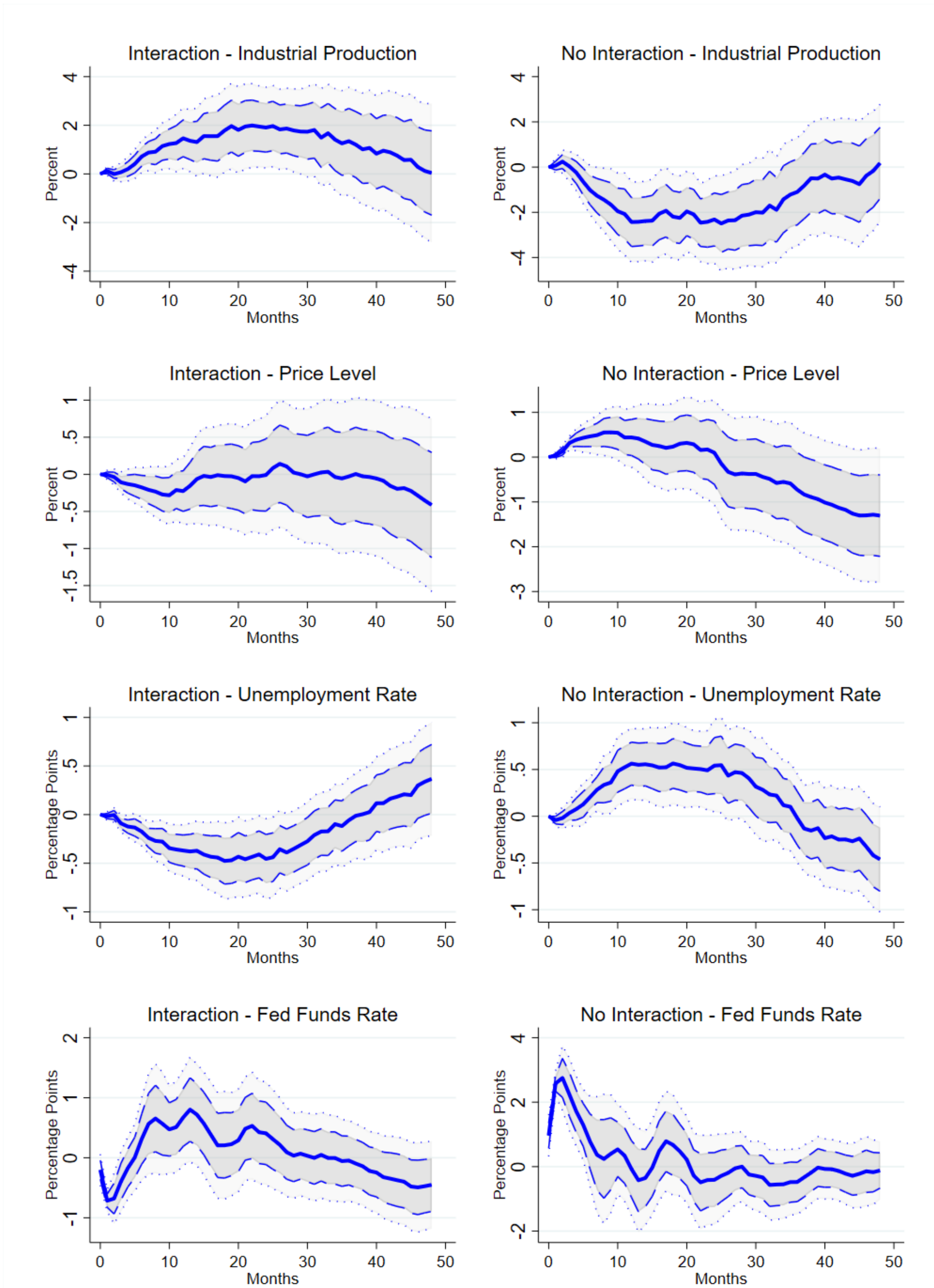
In the main specification, I interacted the monetary policy shock with duration-to-GDP divided by its standard deviation directly. This is appropriate in this context as duration-to-GDP has a well specified meaning as the amount of insurance public debt maturity is providing to the government. However, one might be interested in checking the effects across regimes, across a high and a low duration-to-GDP regimes. In order to do this, in this section, I apply the smooth transition local projection method to my setting. This method was used to estimate the effects of fiscal policy (Auerbach and Gorodnichenko, 2017, Gorodnichenko and Auerbach, 2013, Ramey and Zubairy, 2018) and monetary policy (Tenreyro and Thwaites, 2016) depending on whether the economy is in a recession or expansion. The benefits of a non-linear local projection approach are the same as in the baseline, whereby we can test the effect of a shock in a given state/regime, without restricting the regime to keep staying constant. This method employs a smooth increasing transformation of the state variable of interest, duration-to-GDP in the previous month in this paper, as an interaction term. I follow Granger and Terasvirta (1993)

Figure D.13: Local projection regressions with long debt above 5 years over GDP for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of long debt above 5 years GDP. The first column shows the interaction term of the shock with long debt above 5 years over GDP, the second column shows the shock term not interacted. Each row shows a different LHS variable.

Figure D.14: Local projection regressions with deviation of duration-to-GDP from theoretical duration of one quarter debt



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the interaction term of the shock with the deviation of actual Duration over GDP from the theoretical duration of all debt being one quarter, the second column shows the shock term not interacted. Each row shows a different LHS variable.

as the aforementioned papers, and employ a logistic function on the standardized variable:

$$F(Z_t) = \frac{\exp\left(\theta \frac{Z_t - \bar{Z}}{std(Z)}\right)}{1 + \exp\left(\theta \frac{Z_t - \bar{Z}}{std(Z)}\right)}$$

Where θ controls the speed of transition from one regimes to the other. The reduced form specification with duration-to-GDP becomes:

$$y_{t+h} = \beta_{0,h}^{STLPM} + \beta_{1,h}^{STLPM} Shock_t + \beta_{2,h}^{STLPM} Shock_t F(DurGDP_{t-1}) + \beta_{3,h}^{STLPM} (L)' controls_t + \varepsilon_{t+h} \tag{D.1}$$

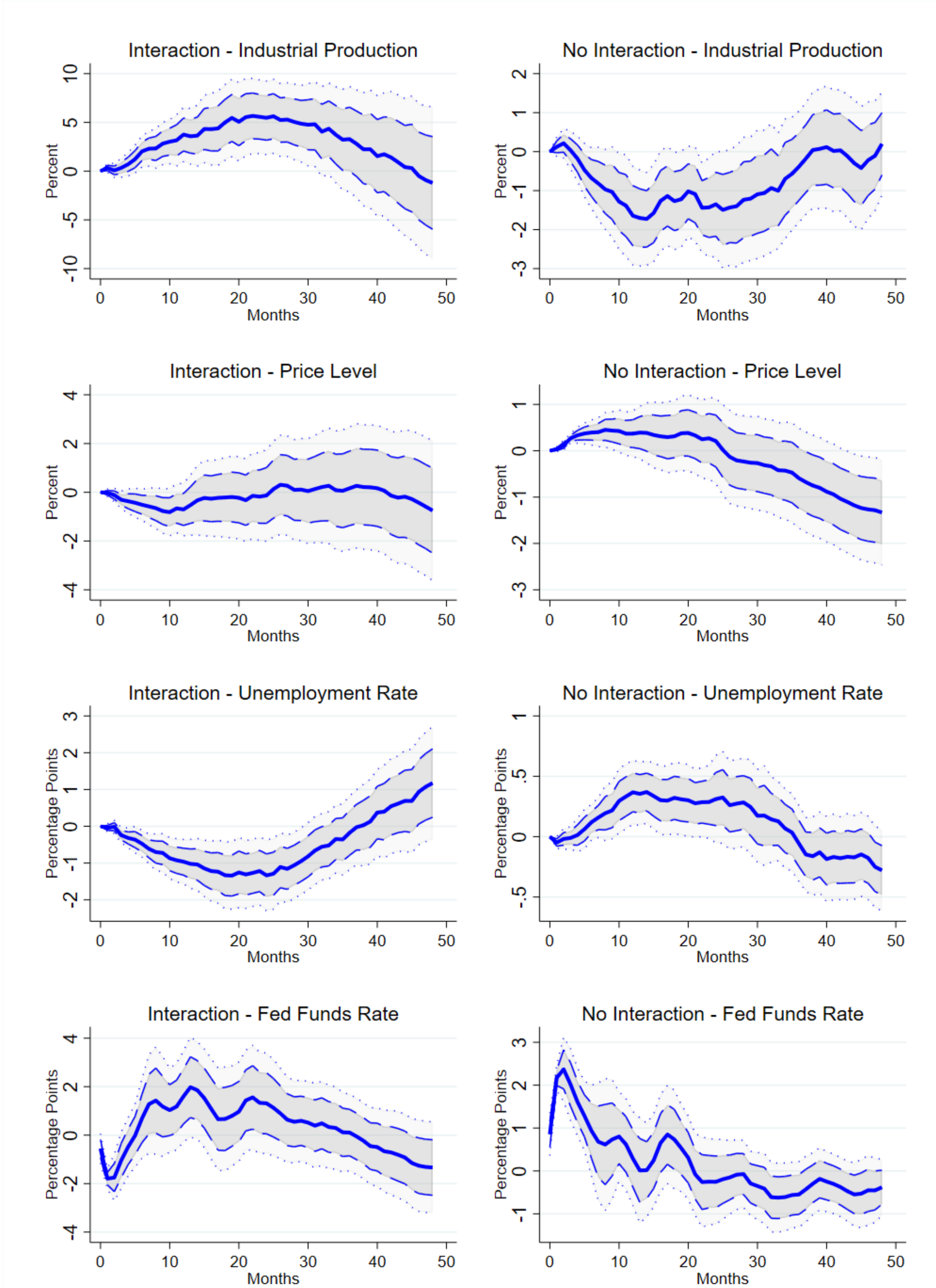
As $F(DurGDP_{t-1})$ is increasing and bounded between zero and one, we can interpret $\beta_{1,h}^{STLPM}$ as response to a contractionary monetary shock on y_{t+h} in a low duration-to-GDP regime, and $\beta_{2,h}^{STLPM}$ as the differential impact when we move from a low to a high duration-to-GDP regime. I follow Tenreyro and Thwaites (2016) and set $\theta = 3$[53].

Figure D.15 presents the results of this experiment. Under a low maturity regimes we still find the same results as in the baseline, monetary policy is more contractionary than on average, at peak the shock reduces industrial production by 1.7 percent. This coefficient has a lower magnitude than the baseline response under a hypothetical overnight debt. The reason is that in this in the baseline we are extrapolating to a hypothetical overnight debt but we never observe it. On the other hand, in this exercise we are looking at the lowest observed values. If we turn to the interaction term, that is, if we move from the low duration-to-GDP regime to the high duration-to-GDP regime we see a coefficient at peak of 5%, which implies that a substantially lower effect of monetary policy on industrial production. This does not imply that the overall effect turns positive under the high duration-to-GDP regime, if we perform the same test as in section D.4 we find a p-value below 0.05 only for the first month, due to the activity puzzle.

The other results are remarkably similar to the baseline as well. The effect on prices is the same across maturity regimes with a reduction in line with the average linear results. Unemployment mirrors industrial production, being relatively lower under the high duration regime to GDP. Finally, also the Fed funds rate responds similarly, with a smaller effect on the first few months if we move from the low to the high duration-to-GDP regime, with the later response being quite similar to the low duration-to-GDP regime.

[53]If we use $\theta = 1.5$ as Auerbach and Gorodnichenko (2017) the results are very close.

Figure D.15: Smooth transition local projection interaction regressions for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and $F(DurGDP_{t-1})$, the smooth logistic transformation of the lag of duration-to-GDP. The speed of transition parameter θ is equal to 3. The first column shows the interaction term of the shock with $F(DurGDP_{t-1})$, the second column shows the shock term not interacted. Each row shows a different LHS variable.

D.11 Results without Recursiveness Assumption

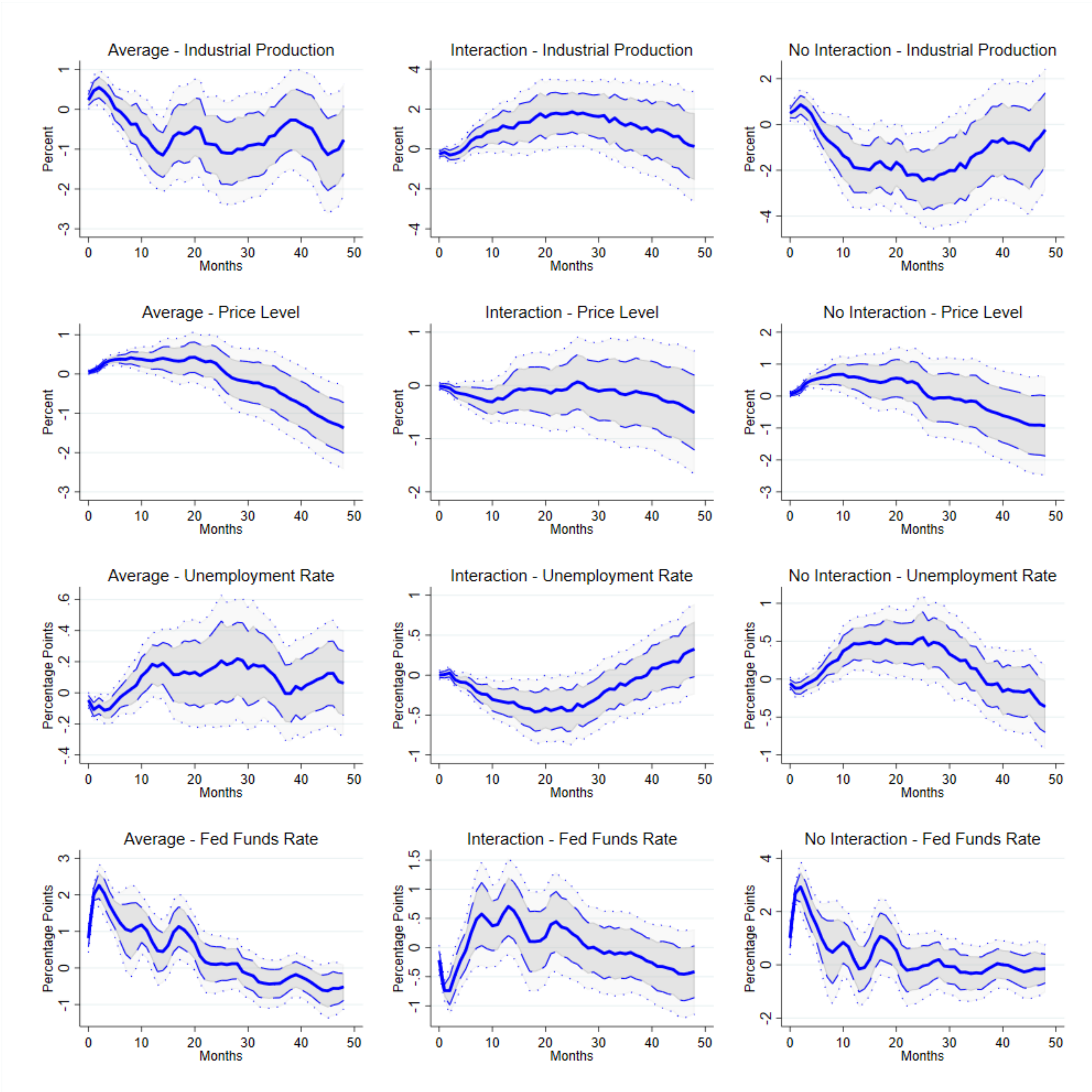
The recursiveness assumption is the assumption that monetary policy cannot affect contemporaneously real variables as industrial production or monetary variables as inflation due to stickiness and lag in action by economic agents. In a univariate local projection setting, the recursiveness assumption is implemented by adding the contemporaneous variables for all the variables which cannot be affected in the same period by the monetary policy shock; in my specification these are industrial production, the price level, and the unemployment rate. In the case without external instruments, it is equivalent to identifying monetary policy shocks with a Cholesky decomposition in a VAR as Christiano, Eichenbaum and Evans (1999) do. When external instruments, such as a narrative or a high frequency instrument, are present, this assumption is not necessary to identify monetary policy shocks but it used to sharpen the identification, as Romer and Romer (2004). Ramey (2016) presents the baseline results with the narrative instrument with the recursiveness assumption and discusses the impacts of not imposing it. In her local projections without interaction terms, she finds that not imposing the recursiveness assumption yields an activity puzzle on industrial production, that is industrial production increases on impact following a monetary policy shock. Furthermore, the price puzzle becomes more pronounced without the recursiveness assumption.

We now move to the specifications with the interaction term of the monetary policy shock with duration-to-GDP presented in Figure D.16. This figure shows the same results as in Figures D.1 and 2 without the recursiveness assumption. The regression is the same, with the exception that contemporaneous controls for industrial production, the price level, and the unemployment rate are not present anymore. In the first and third columns, we can see the same phenomena described by Ramey (2016); there is an activity puzzle for industrial production and unemployment and a marked price puzzle. However, the conditional effect of having long vs short debt are almost the same as in the baseline. When duration-to-GDP is higher monetary policy lowers output less (first row, second column) and increases unemployment less (third row, second column). Moreover, there does not seem to be any differential effect on inflation.

D.12 Results with Recursive Identification

In this subsection I add an additional identification strategy for the monetary policy shock in the spirit of Christiano, Eichenbaum and Evans (1999). The identifying assumption is that monetary policy cannot have an impact on real variables and prices on the same period as the shock happens, but only on the following months. In the context of a univariate local projection this is achieved by including the contemporaneous control for industrial production, the price level, and the unemployment rate. Notice that, the interaction is between the measure of the monetary policy and the lagged value for duration-to-GDP. This implies that the identification

Figure D.16: Local projection regressions without recursiveness assumption for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed without the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration-to-GDP. The first column shows the average effect. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

hurdles specific to the interaction term are the same as in the case with external instruments. Consequently, if one is willing to believe the Cholesky-like recursive identification we interpret the results as in Figure 2. I implement this identification by running the un-instrumented version of (3):

$$y_{t+h} = \beta_{0,h} + \beta_{1,h}\Delta i_{t,t-1} + \beta_{2,h}\Delta i_{t,t-1}DurGDP_{t-1} + \beta_{3,h}(L)'controls_t + \varepsilon_{t+h}$$

Where controls include the lagged value of duration-to-GDP, the first two lags of the Fed funds rate, and the first two lags and the contemporaneous value for industrial production, the price level, the commodity price index, the unemployment rate.

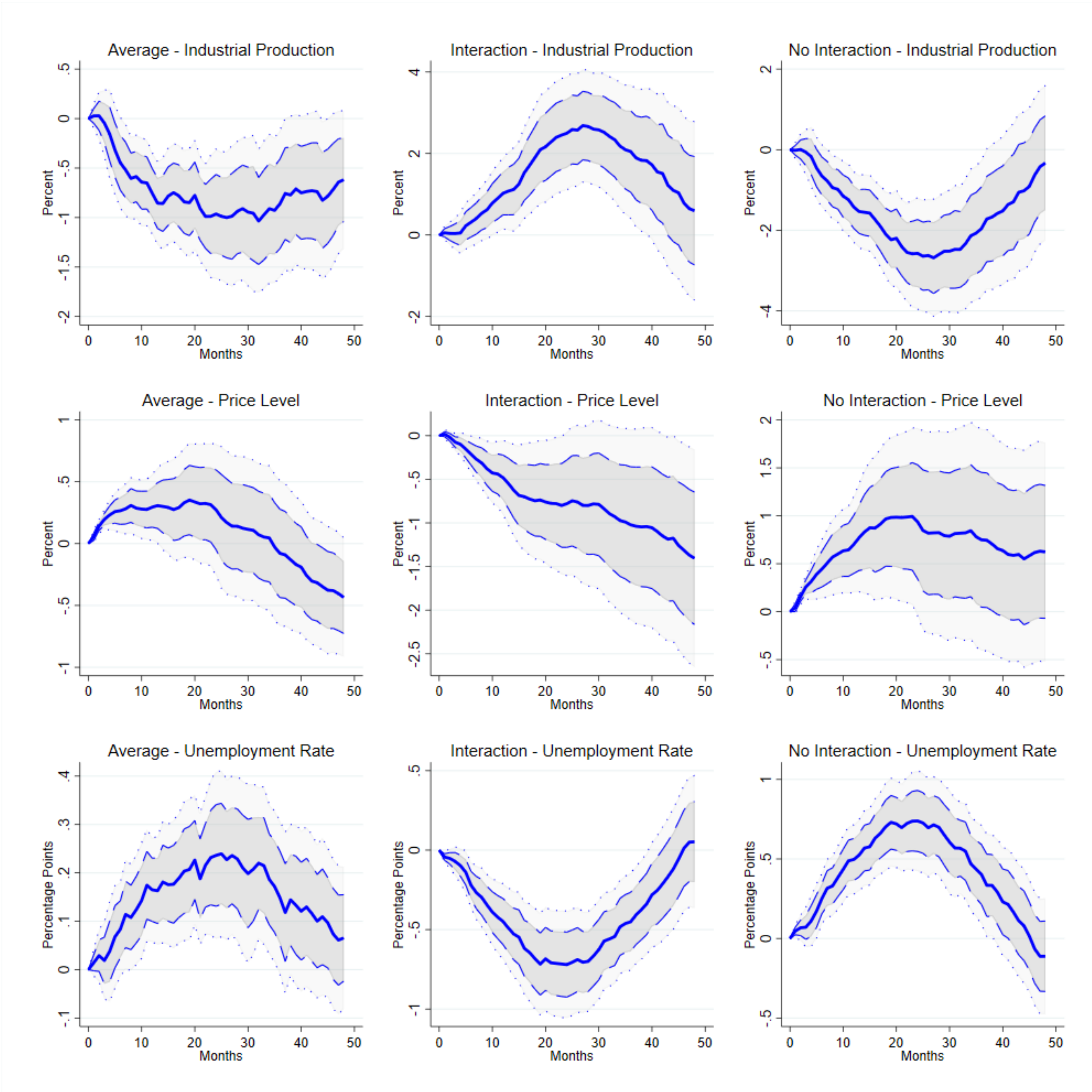
Figure D.17 presents the IRFs with the recursive identification in the same sample as in Figure 2. Even with this identification we can see a strong mediating effect of the maturity structure on the transmission of monetary policy. The higher the duration-to-GDP, the lower the contractionary effect of monetary policy on industrial production and unemployment. At peak, having a standard deviation of duration-to-GDP more lowers the impact of monetary policy by 2.5%. If all debt was overnight, monetary policy would be stronger and would reduce industrial production at almost -3%. Both the interaction term and the no-interaction terms are precisely estimated for industrial production and unemployment, even more than in the external instrument case. The results for the price level are counterintuitive, both for the conditional results and the average linear ones shown in the first row. On average, and in the case of short debt in the conditional case, we see a very long lasting price puzzle, that is, a contractionary monetary policy shock increases prices significantly for 2 years and only then starts to decline. For this reason, I would refrain to interpret the results also on the interaction. There we see a decline, although it is almost never significant.

Figure D.18 presents the same IRFs on a longer sample that goes from 1959m7 to 2013m1. We can perform the regressions on a longer sample as well as we are not limited anymore by the monetary policy shock sample. We can see that the results are virtually unchanged from the restricted sample. We find a strong mediating effect of monetary policy on real variables but find a counterintuitive price puzzle for inflation which does not end even as the horizon ends. Notice that this prize puzzle persists if we include more lags of dependent variables. In an additional exercise I re-run the same specification with 6 lags, with results virtually unchanged. These results are available upon request.

D.13 Results with Original Romer and Romer Shock

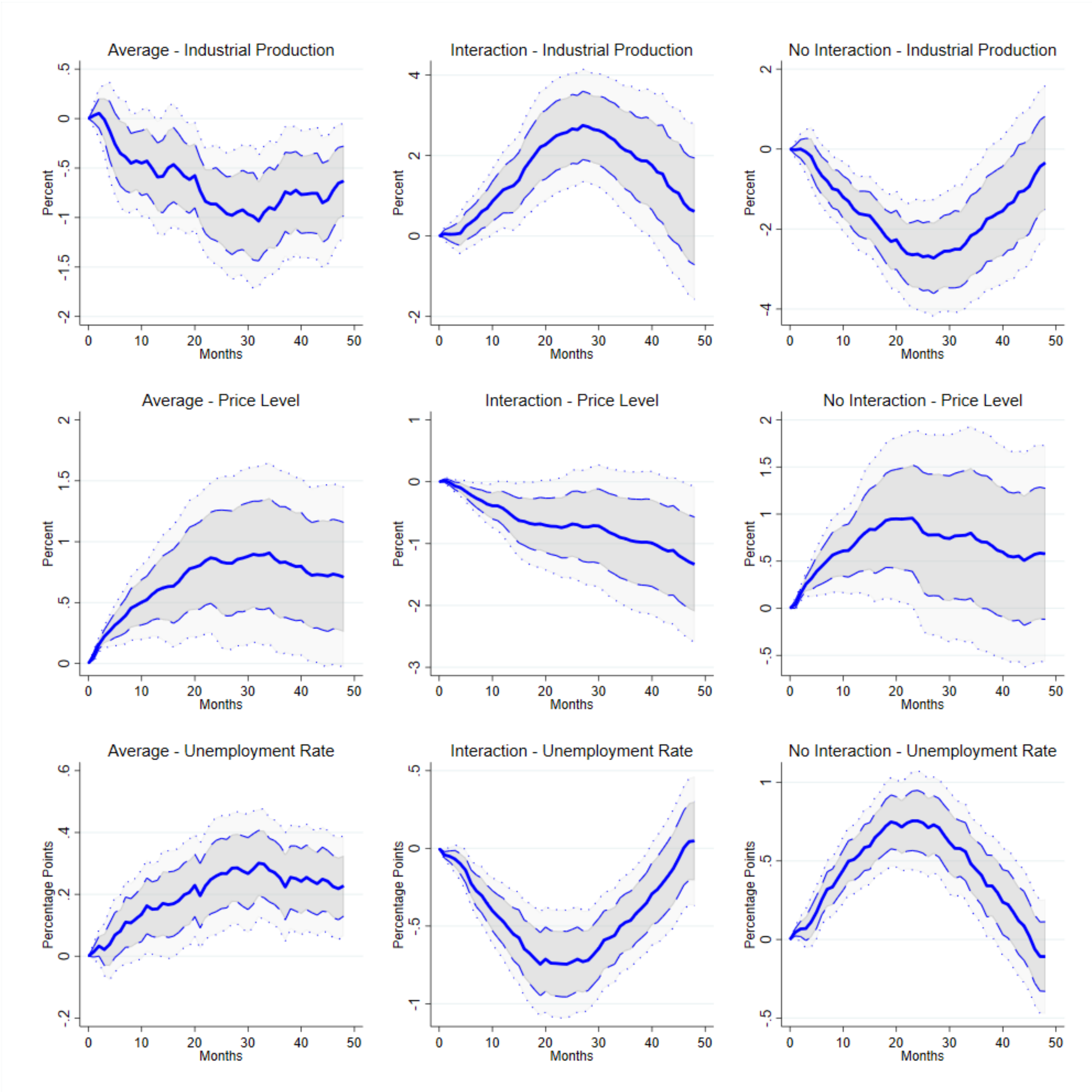
The baseline specification shows the results with the extended sample for the Romer and Romer (2004) by Yang and Wieland (2015) in order to exploit a longer time series variation. In this subsection, I show the results with the original monetary policy shock measure by Romer and

Figure D.17: Local projection regressions with recursive identification for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the recursive identification. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The interaction regression include also and one lag of duration-to-GDP. The first column shows the average linear regression. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

Figure D.18: Local projection regressions with recursive identification for the US with a longer sample



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1959m7 to 2013m1 with US data. Identification of the monetary policy shock is achieved with the recursive identification. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The interaction regression include also and one lag of duration-to-GDP. The first column shows the average linear regression. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

Romer (2004). The sample goes from 1969m1 to 1996m12.

Figure D.19 presents the results of this experiment. The overall patterns are the same as in 2, simply estimated with lower precision, possibly due to the lower sample size. In the first row we can see how monetary policy is less effective on industrial production the higher the duration-to-GDP. At peak, monetary policy is about 2% less effective on industrial production when public debt has a one standard deviation higher duration-to-GDP. This number is similar to the 2% peak in 2. Interestingly, the peak in the baseline specification appears at around two years after the shock, but here it appears at the fourth year. Similarly, the response of inflation does not appear to differ depending on whether debt has a higher or lower duration. When debt is more long term, unemployment is relatively lower following a monetary policy shock and; on impact, interest rate increase by less, although the impact is short lived and quantitatively small.

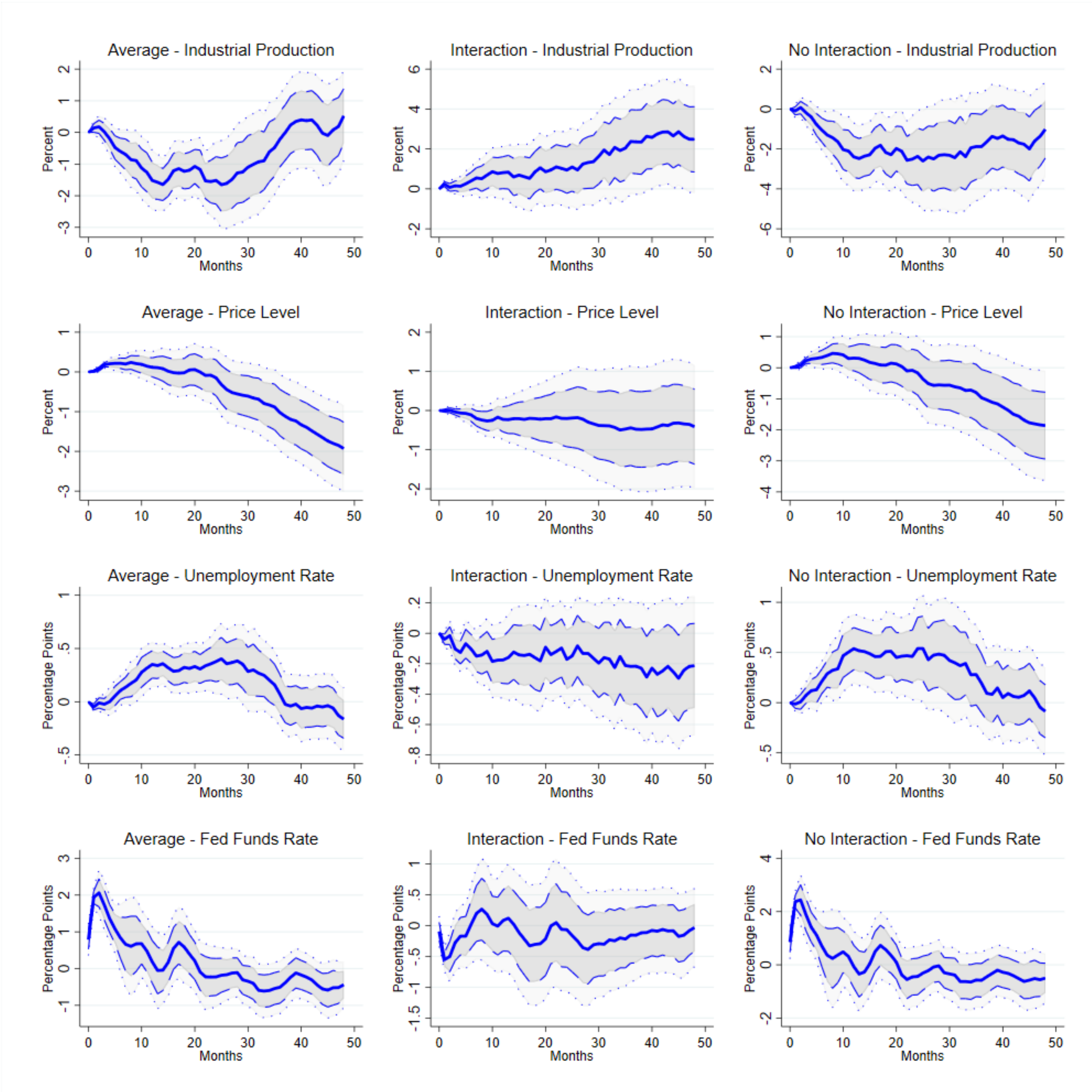
Overall, using the original Romer and Romer narrative measure to identify the monetary policy shock brings similar results as to the updated measure.

D.14 Results with High Frequency Identification

High frequency identification has been used extensively in recent work to identify monetary policy shocks. The key idea is to use changes in federal funds future prices around policy announcements to identify monetary policy shocks that are orthogonal to the information set of financial market participants. The benefit over the narrative method is that, when the FED pursues forward guidance, there are changes in Fed funds futures which are orthogonal to past Greenbook forecast but that might be already anticipated by economic agents. These changes would not be picked up by the high frequency identification scheme. This advantage, together with the exploration of the information channel of monetary policy has contributed to the wide usage of this method in recent research, as Kuttner (2001), Gurkaynak, Sack and Swanson (2004), Gertler and Karadi (2015), Gerko and Rey (2017), Jarociński and Karadi (2020), Miranda-Agrippino and Ricco (2021).

However, this method has a big disadvantage in the context of this study: the sample for which we have data on high frequency future prices. First of all, the sample is too short, as they are available only from the January 1991. This is problematic on its own, especially in a local projection framework, as Ramey (2016) showed that Fed funds futures do not work on their own as instruments on linear average local projections. This is a manifestation of the "power problem" discussed in Nakamura and Steinsson (2018). Moreover, the small time series sample does not allow to have enough variation on duration-to-GDP, which is crucial to estimate the interaction coefficient. Furthermore, the sample with the high frequency identification for monetary policy does not fully overlap with the sample when the identification maturity

Figure D.19: Local projection regressions with original Romer and Romer shock for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 1996m12 with US data. Identification of the monetary policy shock is achieved with the original narrative method by Romer and Romer (2004). Regressions performed without the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The interaction regression include also and one lag of duration-to-GDP. The first column shows the average linear regression. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

structure variation with new narrative account is stronger: that is before QE when central banks entered in the business of affecting the maturity structure of public debt. On the other hand, the sample of the narrative monetary policy identification does.

I try to overcome the problems arsing from the small sample by adding additional structure to these shocks. I extract the high frequency shocks directly from the proxy-VAR ran by Gertler and Karadi (2015), in this way, the estimated structural shock is present from July 1980 to June 2012. The reason for which the sample is longer comes from having a longer estimation sample and a shorter identification sample in the proxy-VAR. The estimation sample is the sample under which the reduced form VAR is ran (e.g. $Y_t = AY_{t-1} + u_t$) which is longer and goes from July 1979 to June 2012. This yields a set of estimated residuals ($\hat{u}_t$) July 1980 to June 2012. In the identification sample, from January 1991 to June 2012, one recovers the mapping from the reduced form residuals to the structural shock by using the high frequency proxy in a set of IV regressions. This mapping can be used on all the sample for which we have the estimated residuals to obtain an estimate for the structural monetary policy shock[54].

This structural shock extracted from a VAR is identified up to a scaling I run these specifications only with the LP-IV specification that easily solve this issue, similarly to Cloyne et al. (2018). Moreover, using the extracted shock as an instrument is beneficial as generated instruments do not suffer from the problems related to generated regressors (Pagan, 1984) as discussed by Wooldridge (2010).

In order to be comparable with the other regressions of this paper and with the Gertler and Karadi (2015) paper I use the following specification for the results presented in Figure D.20. All the regressions have the first two lags of: industrial production, the price level, the unemployment rate, the commodity price index, the unemployment rate, the one year government bond rate, the excess bond premium, the mortgage spread, and the 3 month commercial paper spread. I employ the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. The regression include one lag of duration-to-GDP. The instrumented variables are the change in the one year government bond rate and the interaction between the one year government bond rate change and the lagged duration-to-GDP. The instruments are use the structural shock discussed above.

Figure D.20 presents the results for this exercise. The first column presents the linear average results, without duration-to-GDP, and we can see that the monetary policy shock extracted from the proxy-VAR can replicate the proxy-VAR results for industrial production and unemployment. On average, a monetary policy shock that increases the one year government bond rate lowers industrial production by 1% and increases unemployment by more than 0.5

[54] IRFs are very similar if we apply the same methodology with high frequency instruments that control for the central bank information effect as Jarociński and Karadi (2020) or Miranda-Agrippino and Ricco (2021), instead of the Gertler and Karadi (2015) instrument. Results are available upon request.

percentage points. On the other hand, it still has problem to replicate the results on the price level. The price level is reduced by -0.25%, but the effect is never statistically significant. These results indicate that for real variables, the reason for the discrepancy in results between the proxy-VAR and the LP Ramey (2016) found is likely due to the shorter sample employed with the LP-IV with the futures used directly.

The second and third column of Figure D.20 present the conditional effects. On the first row we can see that monetary policy is less effective on reducing industrial production when duration-to-GDP is higher. At peak, the effect is attenuated by 2% when duration-to-GDP is one standard deviation higher. This coefficient is remarkably close to the baseline coefficient identified with the narrative method. Interestingly, the peak happens at the end of the horizon rather than at the middle as with the narrative method. The effect of a monetary policy shock when all debt is overnight tends to -2%, indicating a stronger contractionary effect with short debt. Again, the peak happens later than under narrative identification, but the magnitude is the same. The second row shows the impact on prices. Here a word of caution is warranted, this method with the proxy-VAR shocks did not work well in the linear average results, so we have to interpret the conditional results with a pinch of salt. These would point to a relatively lower inflation response under a higher duration-to-GDP and even a positive response under a overnight debt scenario. Finally, in the third row we can see how unemployment is specular to industrial production. Unemployment increases relatively less when duration-to-GDP is higher, by 20 basis points. As with industrial production the peak happens relatively later in the horizon.

Overall, the high frequency estimation points to a similar role for the duration of debt to GDP on the transmission of monetary policy. Monetary policy is less effective on reducing output when duration-to-GDP is relatively higher.

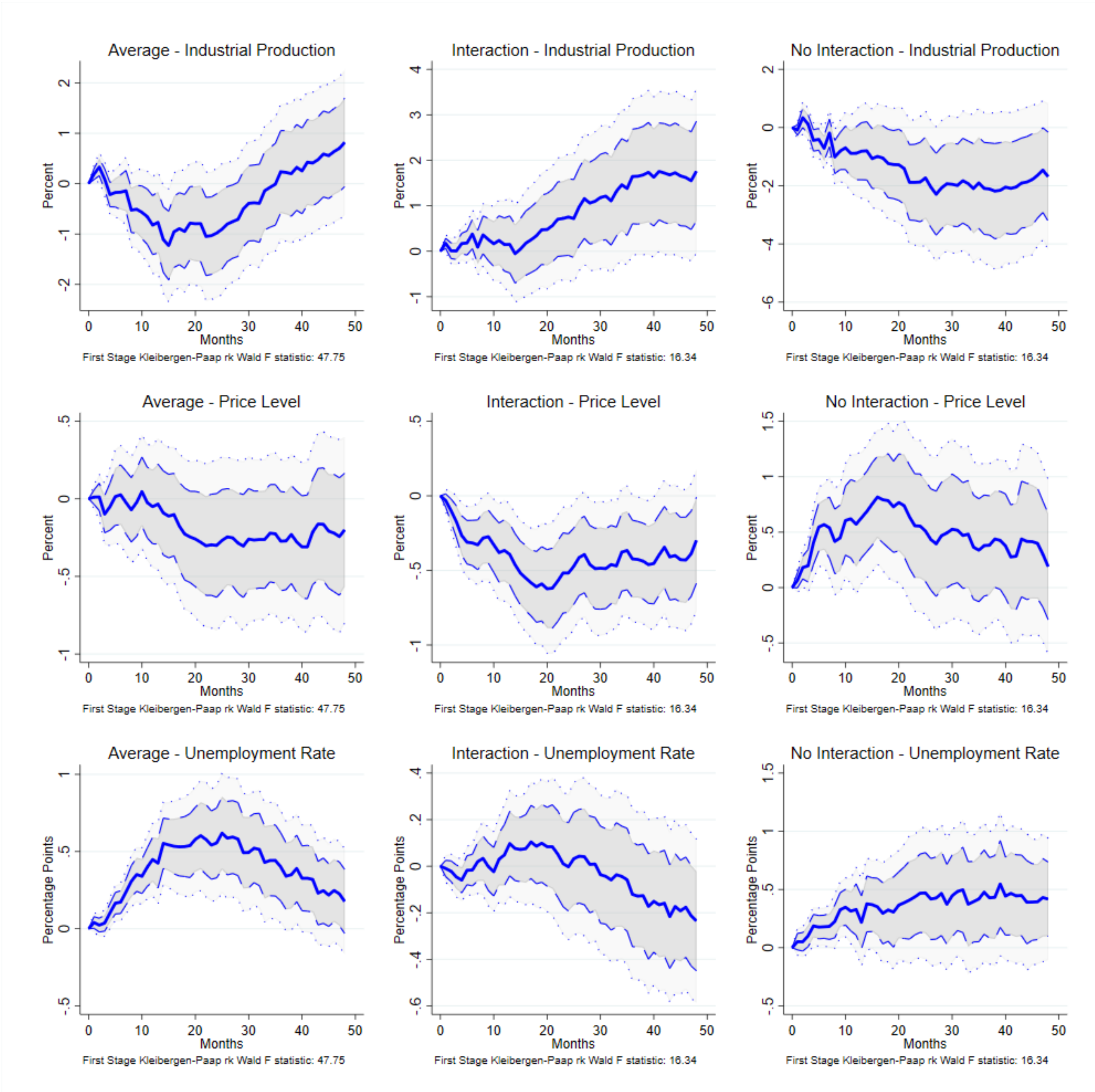
D.15 Results on Possible Endogeneity of the Maturity Structure

A possible concern with the results is that the maturity structure is endogenous and we are picking up a spurious relationship. This could be the case if monetary policy is more effective on output when duration-to-GDP is low for reasons that do not hinge on the maturity structure of public debt. In this section, I show that this is not likely to be a problem by examining reverse causality, possible confounding factors, and by using an instrumental variable approach.

D.15.1 Reverse Causality

Reverse causality, i.e. the debt management authority chooses a lower maturity when monetary policy is more effective on output, is unlikely to be a concern. The debt management office would be choosing to increase the interest rate risk in public debt exactly when interest rate

Figure D.20: Local projection regressions with high frequency identification for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1980m7 to 2012m6 with US data. Identification of the monetary policy shock is achieved with the structural monetary policy shock extracted from the Proxy-VAR of Gertler and Karadi (2015). The instrumented variables are the change in the one year government bond rate and the interaction between the one year government bond rate change and the lagged duration-to-GDP. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, the one year government bond rate, the excess bond premium, the mortgage spread, the 3 month commercial paper spread, and one lag of duration-to-GDP. The first column shows the linear average response to a monetary policy shock. This specification presents the same regressions without the interaction terms and the lag of duration-to-GDP. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

changes have stronger effects on output and tax receipts. It is hard to find a rationale for this: if anything, they should be pushing to lower interest rate risk by lengthening the duration of debt in these periods. Moreover, and very importantly reverse causality is unlikely to be a concern due to the institutional details of debt management: debt management authorities take their maturity decisions at a frequency which is much lower than the one at which monetary policy can affect the economy and in the US legal and political constraints prevented the Treasury from freely choosing the maturity of public debt in the sample under study. In Appendix A, I provide a detailed narrative analysis of this claim for the US Treasury.

D.15.2 Confounding Factors

Alternatively, there could be an omitted variable that drives both a low duration of public debt and a high effectiveness of monetary policy on output. Possible candidates include whether the economy is in a recession, the default risk in the economy, the level and slope of the yield curve, and demographic trends. In Tables D.2 and D.3, I present how these measures correlate with the baseline measure of duration-to-GDP at market value and with Macaulay duration at face value which highlights the maturity structure and it is not mechanically correlated with interest rates.

Tenreyro and Thwaites (2016) show that in recessions monetary policy is less effective on output, so if public debt was longer maturity in recessions, this could confound the results. In columns 3 and 7 of Table D.2, I show that the opposite is true, public debt has shorter maturity during recession, meaning that this phenomenon cannot be the source of the results of this paper.

Another possibility is that when there is higher default risk in the economy the government cannot issue longer debt and monetary policy can affect the economy more as the balance sheet of financial intermediaries is strained. However, in columns 4 and 8 of Table D.2, I find that the opposite is true, a higher default spread in corporate bond markets is associated with longer maturity debt, so that also this hypothesis cannot explain the results.

The level and slope of the yield curve could affect the Treasury's choice of maturity and the effectiveness of monetary policy. When the level of interest rate is low, the maturity of public debt is generally higher, as it can be seen in columns 1 and 5 of Table D.2. However, it is unclear a priori if monetary policy should be more or less effective in periods of low interest rates. On the one hand, low interest rates can be associated with liquidity traps when monetary policy is less effective; on the other hand, when interest rates are at a low level, a given change change in interest rates has stronger effects on collateral values implying potentially stronger effects of monetary policy. We check directly if monetary policy is more or less effective on output when interest rates are low, by running the baseline local projection regression (2), with the level of interest rates in the previous month as the interaction term. This regression shows there are no strong conditional effects of monetary policy depending on the interest rate levels, the figure is available upon request.

The slope of the yield curve could also be a confounding factor. A flat yield curve is generally associated with recessions, which are associated with lower effectiveness of monetary policy (Tenreyro and Thwaites, 2016). If the treasury takes advantage of the flat yield curve to increase the maturity structure, then a longer duration would be associated to weak effects of monetary policy that are independent of the proposed channel. However, in columns 2 and 6 of Table D.2, we see the opposite pattern: a flat yield curve is actually associated with high duration of public debt! This implies that this is not likely to be a confounding factor[55].

Demographic trends could be driving both the strength of monetary policy and the maturity structure of public debt. As societies grow older the effectiveness of monetary policy could be altered, but it is ex-ante ambiguous how. Monetary policy could become less strong as argued by Wong (2021) and Cloyne, Ferreira and Surico (2019) as older households do not hold mortgages and are less likely to be liquidity constrained. On the other hand, monetary policy could become stronger as argued by Berg et al. (2019) if wealth effects are quantitatively important or if older households consume relatively more in sectors with sticky prices. In the first case, if an older population is associated with longer maturity debt, as people demand longer debt for retirement, this could be a possible confounding factor that could explain the results. However, in the data we see opposite patterns in the US and in the UK. In Table D.3, I show how the age dependency ratio is positively correlated with duration-to-GDP in the US but negatively in the UK. This implies that demographic trends are unlikely to confound the results.

Finally, the external validity of the US results with UK data can help to shed light on potential confounding factors through a direct comparison. Many real and financial macroeconomic variables are strongly correlated across countries (see Miranda-Agrippino and Rey, 2020), therefore, the correlation of duration-to-GDP across the two countries is a useful metric to look at. If the correlation was positive, one could worry that a common factor was driving both variables. Table D.4 presents the correlation between the US and the UK duration-to-GDP at market (face) value on the first (second) column. The relationship is negative and statistically significant for both measures, implying that it is unlikely that a common confounding factor drives them both.

[55] Note that the positive correlation of slope and maturity is of independent interest: it implies that in equilibrium the treasury does not tilt its maturity position to take advantage of the flat yield curve. When the treasury has issued more long term debt, its relative price, compared to short term debt is higher, in line with the preferred habitat theory of Vayanos and Vila (2021).

Table D.2: Duration measures regressions

VARIABLES	(1) $DurGDP$	(2) $DurGDP$	(3) $DurGDP$	(4) $DurGDP$	(5) Dur^{FV}	(6) Dur^{FV}	(7) Dur^{FV}	(8) Dur^{FV}
YC Level	-0.0674***				-0.0520***			
	(0.00361)				(0.0114)			
YC Slope		0.127***				0.339***		
		(0.0105)				(0.0312)		
Recession			-0.342***				-0.343**	
			(0.0435)				(0.150)	
Default Spread				0.303***				0.468***
				(0.0251)				(0.0774)
Observations	468	468	468	420	468	468	468	420
R-squared	0.221	0.148	0.091	0.188	0.017	0.134	0.012	0.056

Notes: Robust standard errors in parentheses. The sample goes from 1969m1 to 2007m12 for columns 1, 2, 3, 5, 6, and 7 and from 1973m1 to 2007m12 for columns 4 and 8. The regressions are on US data. The depended variables are duration-to-GDP at market value for columns 1 to 4 and the Macaulay duration at face value for columns 5 to 8. ”YC Level” is the yield curve level measured with the 3 month rate on government debt, ”YC Slope” is the yield curve slope measured with the difference between the 10 years and 3 month rates on government debt, ”Recession” is a dummy for the NBER based Recession Indicator (Fred code: USRECM), and the ”Default Spread” measures the default premium on corporate bonds estimated with the Gilchrist and Zakrajšek (2012) methodology. Each regression also contains a constant.

Table D.3: Duration to GDP in US and UK and Demographic Trends

VARIABLES	(1) $DurGDP_{US}$	(2) Dur^{FV}_{US}	(3) $DurGDP_{UK}$	(4) Dur^{FV}_{UK}
US Dependency Ratio	0.585***	0.941***		
	(0.0102)	(0.0312)		
UK Dependency Ratio			-0.138***	-1.727***
			(0.0307)	(0.0480)
Observations	348	348	348	348
R-squared	0.891	0.465	0.055	0.449

Notes: Robust standard errors in parentheses. The sample goes from 1979m1 to 2007m12. The first two columns are the measures on US data, columns 3 and 4 are the measures on UK data. The first and third columns show the correlation for duration-to-GDP at market value, the second column and fourth show the correlation for Macaulay duration at face value. The Dependency Ratio variable measures the ”Age Dependency Ratio: Older Dependents to Working-Age Population” from the World Bank data (Fred codes: SPPOPDPNDOLUSA and SPPOPDPNDOLGBR) and it is interpolated from annual to monthly frequency. Each regression also contains a constant.

Table D.4: Duration to GDP in US and UK Comparison

VARIABLES	(1) $DurGDP_{US}$	(2) $DurGDP_{US}^{FV}$
$DurGDP_{UK}$	-0.0877**	
	(0.0411)	
$DurGDP_{UK}^{FV}$		-0.478***
		(0.0142)
Observations	348	348
R-squared	0.007	0.544

Notes: Robust standard errors in parentheses. The sample goes from 1979m1 to 2007m12. The left hand side variables are the measures on US data, the right hand side ones are the measures on UK data. The first column shows the correlation for duration-to-GDP at market value, the second column shows the correlation for duration-to-GDP at face value. Each regression also contains a constant.

D.15.3 Instrumental Variable Approach

As an additional check that higher effectiveness of monetary policy under a low maturity regime is not due to confounding factors, I employ an instrumental variable approach for duration-to-GDP. Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood and Vayanos (2014) suggest that the overall stock of government debt is a good instrument for the maturity structure of public debt, with metrics similar to duration over GDP. The argument is that the overall stock of public debt is a good instrument as it is orthogonal to current market conditions since it is due to past government deficits. One can test the relevance assumption with the first stage F-statistic. I follow Greenwood and Vayanos (2014) and use the stock of government bonds, including TIPS, held by the general public at face value, in order to purge the measure from price movements. I employ the baseline LP-IV specification presented in equation (3) and instrument the change in the federal funds rate, the lag of duration-to-GDP, and their interaction, with the narrative monetary shock, the lag of debt to GDP, and their interaction. Columns 1 to 3 of Table D.5 presents the first stage, and Figure D.21 present the IRFs.

The first stage is strong with a robust F-Stat at 37.87. Moreover, we can see that the strongest effects, with the highest level of significance, can be found on the diagonal. This implies that the highest predictive power for each variable can be found in its direct instrument counterpart, e.g. for the interaction term in column 2 the only significant instrument is the iteration terms between the monetary policy instrument and debt to GDP. As a robustness check, columns 4 to 6 of Table D.5 show the same first stage regressions without using any macro control. The magnitude, sign, and significance of the coefficients are all very similar and the robust F-Stat is still high at 29.45.

Figure D.21 presents the IRFs from this experiment. The results are very similar in di-

Table D.5: First stage regressions with instruments for interest rates and for duration-to-GDP

VARIABLES	(1) $\Delta i_{t;t-1}$	(2) $\Delta i_{t;t-1} DurGDP_{t-1}$	(3) $DurGDP_{t-1}$	(4) $\Delta i_{t;t-1}$	(5) $\Delta i_{t;t-1} DurGDP_{t-1}$	(6) $DurGDP_{t-1}$
$Shock_t$	1.019***	-0.160	0.143**	1.165***	0.00616	0.107
	(0.242)	(0.193)	(0.0718)	(0.332)	(0.252)	(0.0897)
$Shock_t DebtGDP_{t-1}$	-0.152*	0.493***	-0.0948**	-0.156	0.476***	-0.101*
	(0.0807)	(0.0884)	(0.0414)	(0.103)	(0.107)	(0.0584)
$DebtGDP_{t-1}$	-0.193**	-0.0985	0.969***	-0.0308	-0.0451	0.978***
	(0.0812)	(0.0837)	(0.0526)	(0.0292)	(0.0307)	(0.0184)
Observations	467	467	467	467	467	467
Robust F-Stat	37.87	37.87	37.87	29.45	29.45	29.45
Controls	Recursive	Recursive	Recursive	Minimal	Minimal	Minimal

Notes: Newey-West standard errors in parentheses. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Identification of duration-to-GDP is achieved with public debt to GDP. Public debt is measured with the stock of marketable nominal and inflation linked debt at face value. The depended variables are the instrumented variables in the LP-IV framework; they are the change in the Fed funds rate, the interaction between the Fed funds rate change and the lagged duration-to-GDP, and the lagged duration-to-GDP. Duration to GDP is measured with the marketable nominal debt at market value held by the general public. The first three columns show the first stage regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, these two regressions include the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. Columns 4 to 6 show the same first stage regressions with minimal controls, that is, controlling only for a constant.

rection, magnitude, and significance to the baseline LP-IV results presented in Figure D.5. A contractionary monetary policy shock is attenuated on its impact on industrial production and unemployment when public debt has a longer duration and there is no differential effects on inflation[56].

The analysis of this subsection shows that the endogeneity of the maturity structure is not likely to pose a problem in the interpretation of the empirical results.

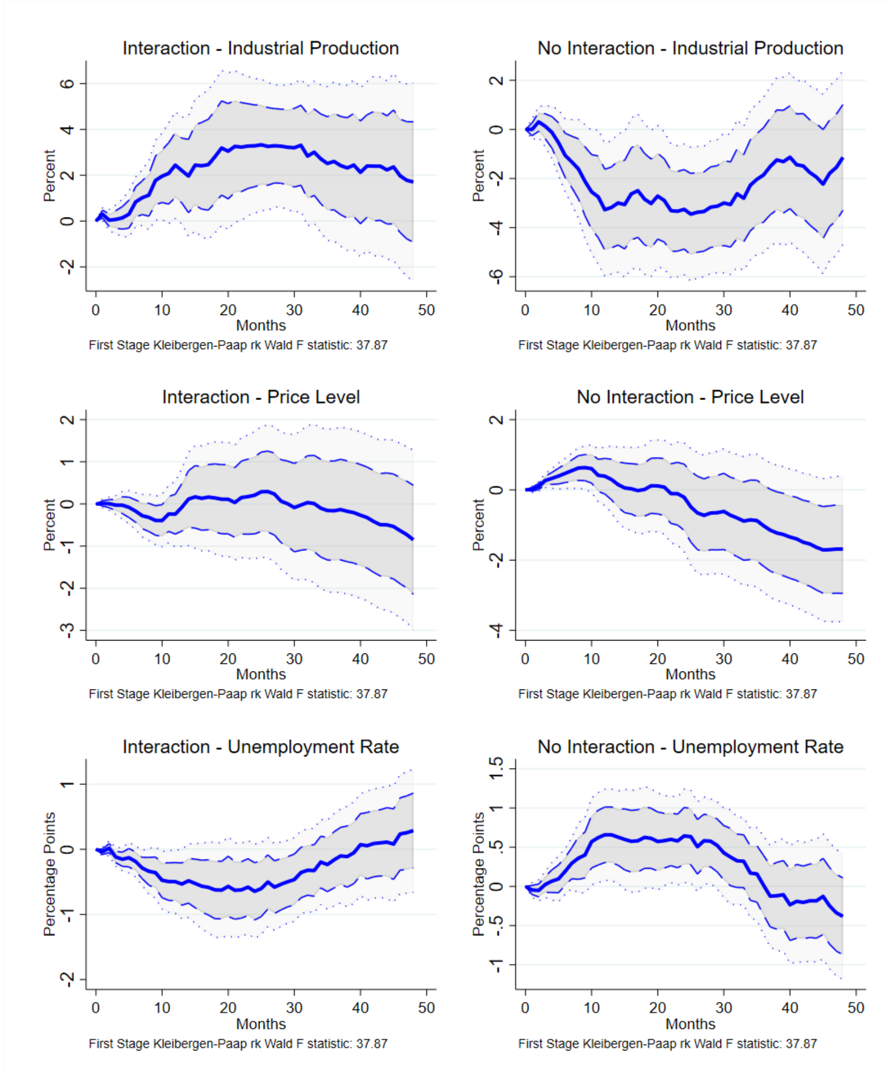
E Further Empirical Results

E.1 Results on Bond Yields and Prices

Monetary policy has strong effects not only on the short end of the yield curve, but also can have effects on the long end of the curve, as argued by Nakamura and Steinsson (2018). This provides a rationale for why a shock to the short rate can matter for valuation of long-debt and how long-debt can provide insurance against such shock. Even if monetary policy affects more strongly short than long rates, it may have large effects on valuation of long debt because debt prices at longer horizons react more than one to one to variations in the interest rate. Take a 10 year zero coupon bond with a yield continuously compounded, its price is: $p_{10} = e^{-10y_{10}}$. Therefore, the derivative of the log of the price to a change in the interest rate is such that an increase in one percentage point in that yield leads to a decrease of 10 percent in the bond

[56]Results are very similar also if the instrument for duration-to-GDP is measured with only nominal debt, with debt at market value, or with debt also held by the government. These IRFs are available upon request.

Figure D.21: Local projection instrumental variable with instruments for interest rates and for duration-to-GDP



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Identification of duration-to-GDP is achieved with public debt to GDP. Public debt is measured with the stock of marketable nominal and inflation linked debt at face value. The instrumented variables are the change in the Fed funds rate, the lagged duration-to-GDP, and their interaction. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate. The first column shows the interaction term of the change in the Fed funds rate with the Duration over GDP, the second column shows the Fed funds rate term not interacted. Each row shows a different LHS variable.

price: $\frac{\partial log(p_{10})}{\partial y_{10}} = -10$, which is much larger than the impact on a short bond price.

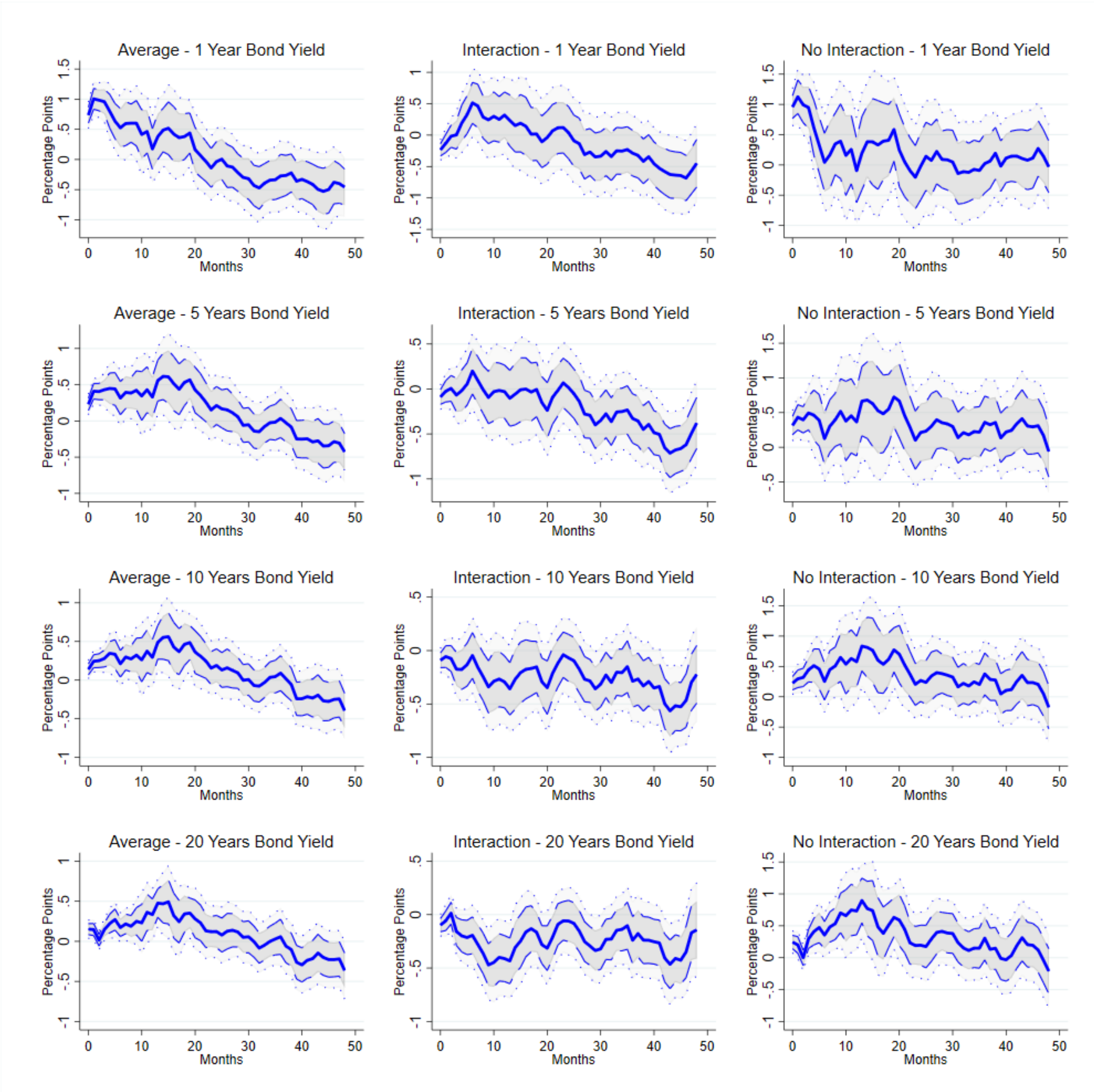
Column 1 of Table E.1 presents the impact on yields at different maturities to a contractionary monetary policy shock.[57] The first row presents the response of the 1 year bond yield, the second of a 5 year bond, the third of a 10 year bond, and the fourth of a 20 year bond. All yields increase with the shorter term yield respond relatively more to a monetary policy shock but with still strong effects on longer maturity yields.

In the first column of Table E.2 we can now see the impact on the log of the corresponding bond prices multiplied by 100. Despite the fact that long maturity bond yields moved less than the short ones, the prices decline a substantial amount. Whereas the price of one year bonds declines by about 1 percent, the price of 20 years bonds declines by 10 percent.

Furthermore, the effects on bond yield and prices seems to be the same irrespective of the whether public debt duration to GDP is low or high. Columns 2 and 3 of figures E.1 and E.2 present the interaction regressions with duration to GDP. Column 2 shows how the interactions terms are not statistically different from zero. This result gives weight to the idea that what matters is the insurance mechanism that long debt provides, but it is not affected by the transmission through the yield curve.

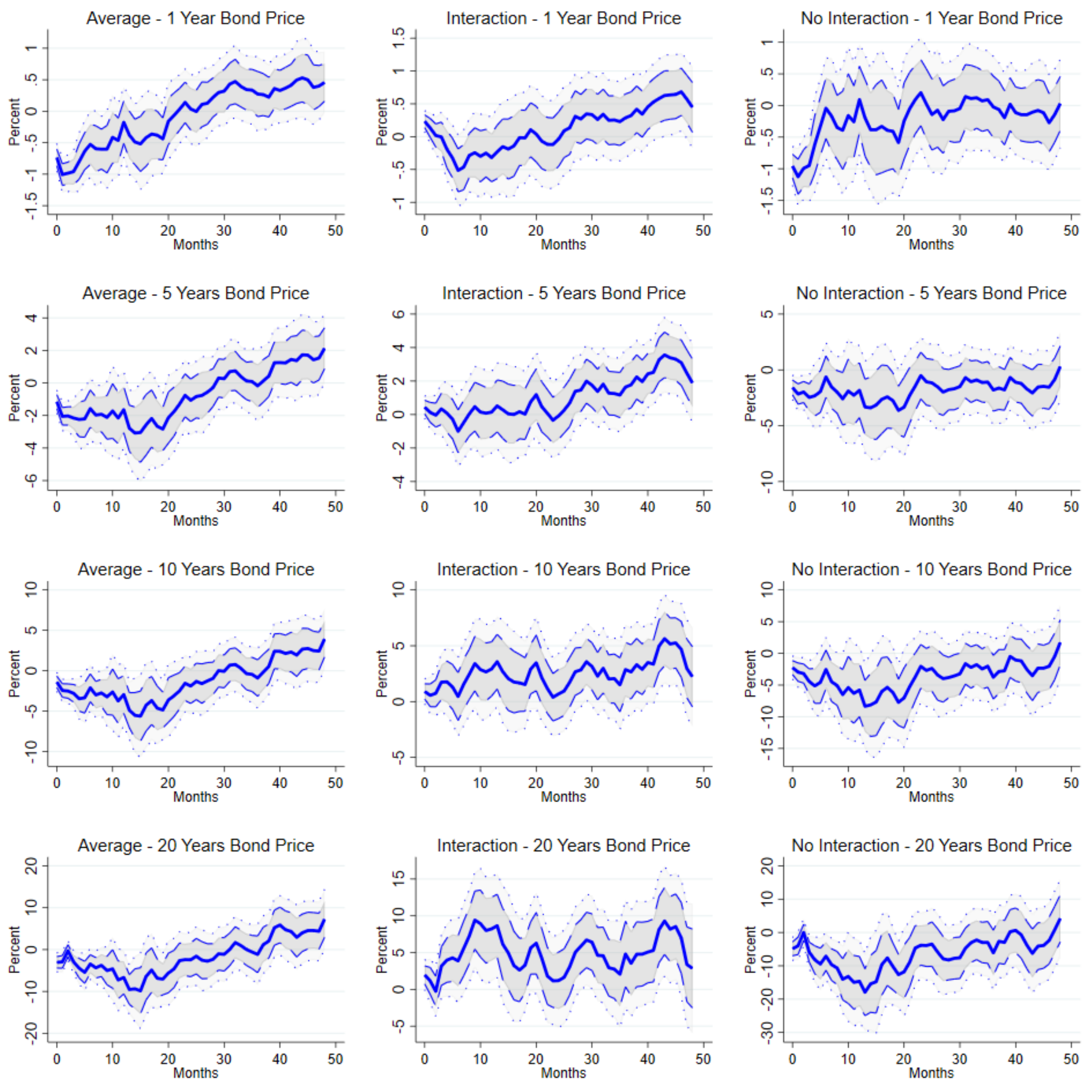
[57] All the tables in this section display the results with controls as in the baseline results with the recursiveness assumption on macroeconomic variables and with two lags of bond yields. Alternative specifications produce similar IRFs and are available upon request.

Figure E.1: Local projection regressions with interaction of duration to GDP for bond yields for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the unconditional response to a monetary policy shock. This specification presents the same regressions without the interaction terms and the lag of duration to GDP. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

Figure E.2: Local projection regressions with interaction of duration to GDP for bond prices for the US



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1969m1 to 2007m12 with US data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, the price level, the commodity price index, and the unemployment rate. In addition, each regression contains the first two lags of industrial production, the price level, the commodity price index, the unemployment rate, and the Fed funds rate and one lag of duration to GDP. The first column shows the unconditional response to a monetary policy shock. This specification presents the same regressions without the interaction terms and the lag of duration to GDP. The second column shows the interaction term of the shock with the Duration over GDP, the third column shows the shock term not interacted. Each row shows a different LHS variable.

F Results with UK Data

F.1 Baseline LP Results without Interaction Term

As in the case of the US, the first step is to present the average effects of monetary policy. For this reason, this subsection presents the LP and LP-IV regression results for the narrative identification from Cloyne and Hürtgen (2016) for the UK.

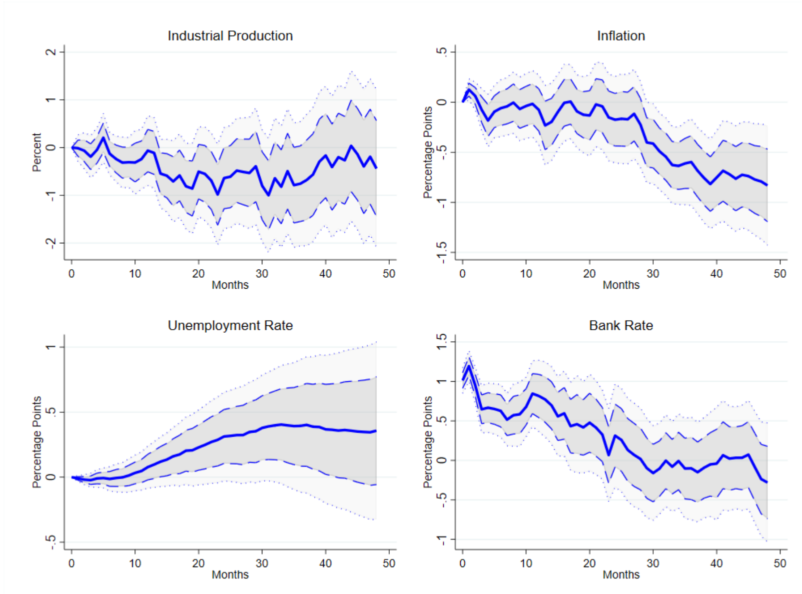
Figure F.1 presents the replication of the results of Cloyne and Hürtgen (2016) for the impact of the monetary policy shock on key macroeconomic variables with a reduced form local projection. The regressions incorporate the recursiveness assumption and have 4 lags of first differences of the log of industrial production, of the year-on-year RPIX inflation (that excludes mortgage payments), of the unemployment rate, of the Bank rate, and of the log of the commodity price index. In addition, each regression adds 48 lags of the monetary policy shock as in Cloyne and Hürtgen (2016). As in that paper, I run the specification in first differences for the the macroeconomic controls and with h-steps ahead differences for the dependent variable ($y_{t+h} - y_{t-1}$). The estimation sample goes from 1979m1 to 2007m12. In the first panel we can see the response of industrial production. The impulse response functions are not very precisely estimated, but we can see how industrial production declines by around one percent 2 years after the monetary policy shock and it reverts slowly to zero at then end of the 4 years window. For inflation, we can can see in the second panels, that the monetary policy shock does not have a strong effect in the first 2 years, but then turns negative and reaches almost one percentage point reduction at the end of the sample. Unemployment, shown in the third panel, behaves more smoothly and it increases by almost half a percentage point 3 years after the shock. Finally, the bank rate increases by one percentage point on impact and then reverts back to the baseline in less than 2 years. Overall, the results point to strong effects of monetary policy, with a delayed impact, especially on inflation. As the US case, the results are quite similar with a local projection instrumental variable (LP-IV) estimation, they are available upon request.

F.2 Results with Duration to GDP

This section presents the baseline interaction results for the UK with a reduced form. Figure F.2 mirrors Figure 2 for the US.

Each regression follows the same specification as in the unconditional regressions with in addition the interaction of the lag of duration to GDP with the measure of the shock and one lag of duration to GDP on its own. In each row of F.2 we can see a different dependent variable. The second column presents the impact of a monetary policy shock in the hypothetical situation when all debt is overnight. The first column presents the impact on increasing duration to GDP

Figure F.1: Unconditional local projection regressions for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable.

by one standard deviation on the effect of a monetary policy shock on the dependent variable. Notice that, the UK has had high average level for duration to GDP throughout the sample as we can see by comparing Figures C.5 for the UK, with Figures 1 for the US, in line with the idea that the UK has the longest maturity of public debt among large economies. This implies that extrapolating the effect of a monetary policy under overnight duration is less informative for the UK than for the US. This implies that for these results it makes the most sense to focus on the interaction term only[58].

The first row of Figure F.2 presents the response of industrial production to a contractionary monetary policy shock. The key result can be found in the first column, monetary policy shocks are attenuated when duration to GDP is higher. An increase of one standard deviation of duration to GDP reduced the contractionary impact of monetary policy by 2% at peak. This effect is economically large, statistically significant, and remarkably close to the US result. Notice that the path of duration to GDP of the UK and of the US is not positively correlated, which gives credit to the idea that we are not picking up something driving both results but a

[58]To overcome the difference in levels, we can compare regimes of historically low duration to GDP with regimes with historically high duration to GDP with a smooth transition local projection method. The UK results are presented in Section F.4 and the US ones in D.10. With that specification the overall magnitudes also under a low maturity regime are very similar.

feature of the maturity stricture of public debt. On the UK estimation sample, from 1979m1 to 2007m12 the correlation between the duration to GDP across the two countries was -0.0830. If we look at the effect of a contractionary monetary policy shock under overnight debt on the second column, we can see how the effect is stronger than in the baseline. However, as overnight debt is not a reasonable comparison for the UK the magnitude of the coefficient is too large.

We now turn to the effects in inflation presented in the second row of Figure F.2. Having a lower or higher level of duration to GDP does not seem to alter the transmission of monetary policy to inflation. The interaction coefficient is not statistically different from zero in any horizon except for a short blip in months 5 and 6 of a small magnitude. If we move to the no-interaction term we see the same pattern as in the unconditional response, being not precisely estimated for the same reason as for industrial production. This result also chimes with the US one.

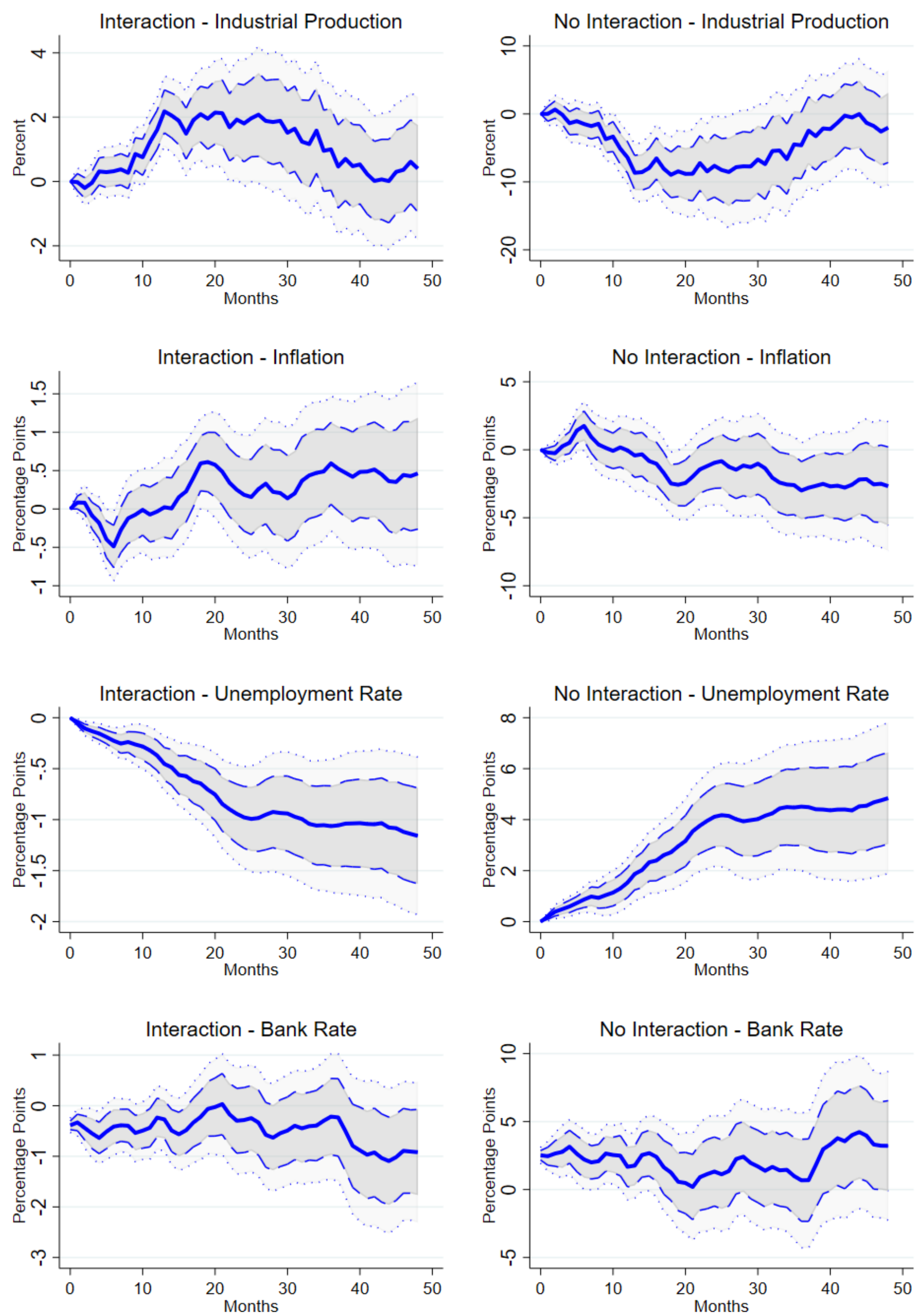
Unemployment, shown in the third row of figure F.2, behaves specularly to industrial production. An increase of one standard deviation in duration to GDP lowers the effect of a contractionary monetary policy shock on unemployment by one percentage point. The effect is stronger than in the US, but is consistent with a higher response of unemployment also unconditionally in the UK. An interesting feature is that we do not seem to converge back to zero for unemployment at the end of the sample, possibly indicating that the effects tend to be longer lived.

Finally, the fourth row of Figure F.2 presents the results for the response of the Bank Rate. In the interaction IRF we can see that there is a mild lower impact of the shock when debt to GDP has a longer duration. This effect is small and short lived, in line with the predictions of the structural model. If we look at the non-interaction effects we can see stronger response, however, the magnitude is relatively high due to the no-interaction coefficient being the comparison with a unlikely overnight debt.

The key take away of this exercise is that, the interaction results are similar in sign and magnitude to the US ones. A higher level of duration to GDP attenuates the contractionary effect of monetary policy on reducing output but does not have an effect on prices. This happens despite the fact that the US and the UK are countries with vastly different duration to GDP, both in level and in time series properties.

Notice that also the UK results are robust to different specifications and debt construction mechanism. The next section shows the same robustness checks as in the US that are feasible in the UK data. The only results to highlight particularly is the one presented in Section F.4 as it allows a more appropriate comparison to the US results.

Figure F.2: Local projection baseline interaction regressions for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Duration over GDP, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

F.3 Additional Empirical Results

This section presents the robustness checks on the baseline specification for the UK. Each result mirrors the US ones presented in Appendix D, which contains a more detailed discussion of each of these specification. Figure F.3 mirrors Figure D.6 for using the Macaulay duration instead of duration to GDP. Finally, Figure F.4 mirrors Figure D.10 by including inflation linked debt in the duration to GDP measure[59].

Overall, all of these specifications support the main finding that long duration debt to GDP attenuates the impact of monetary policy on output, but does not have an effect on prices. It should be noted that we cannot construct the series with public holding of government debt by public entities as the Bank of England, so we cannot distinguish the effect with and without these holdings, as I did for the US. However, this is unlikely to be a problem as the estimation sample ends on December 2007 and the Bank of England started to buy large amounts of public debt only from QE in 2009. Finally, it is particularly reassuring that the results go through when including inflation linked debt, as they are a larger share of the UK debt.

F.4 Results with Smooth Transition Method

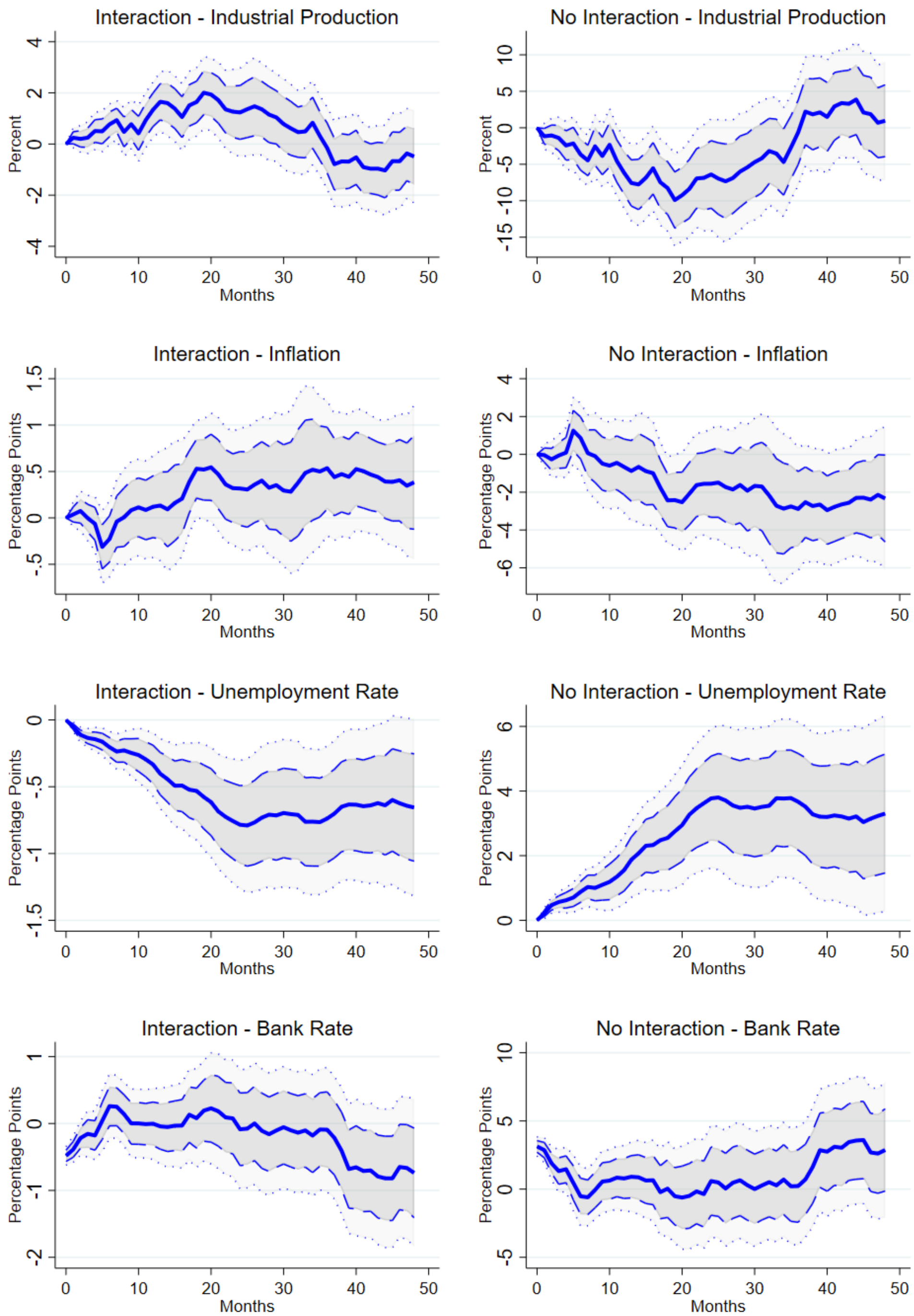
Finally, I replicate the smooth transition local projection method for the UK data as well. This method is particularly well suited for the UK data as, due to the historically high duration to GDP in the UK, the overnight debt comparison I employ in the baseline is less intuitive than in the US. In the smooth transition method, we study the effects of a monetary policy shock under a low duration to GDP regimes and we compare it with the high duration to GDP regimes as shown in equation (D.1) and discussed in section D.10.

Figure F.5 presents the results from this exercise with the baseline specification. If we examine the effect of a monetary policy on industrial production under a low duration to GDP regime we see results which are quite close to the US results: the reduction is about 2% at peak. This shows how using the regime comparison is particularly useful for the UK. The results on the iteration term, that is the difference in effect on output when we move from a low to a high duration to GDP regime, are also in line with the US ones, with a positive coefficient of about 5%. Monetary policy is much less effective on output when the economy is in a high duration to GDP regime.

When we turn to prices we can still a find similar result to the US. Under the low duration to GDP the effect of a monetary policy reduced inflation by about 1%. If we move to a high duration to GDP regime we still find no statistically significant difference. Unemployment mirrors industrial production, and the Bank rate increases relatively less during a high duration

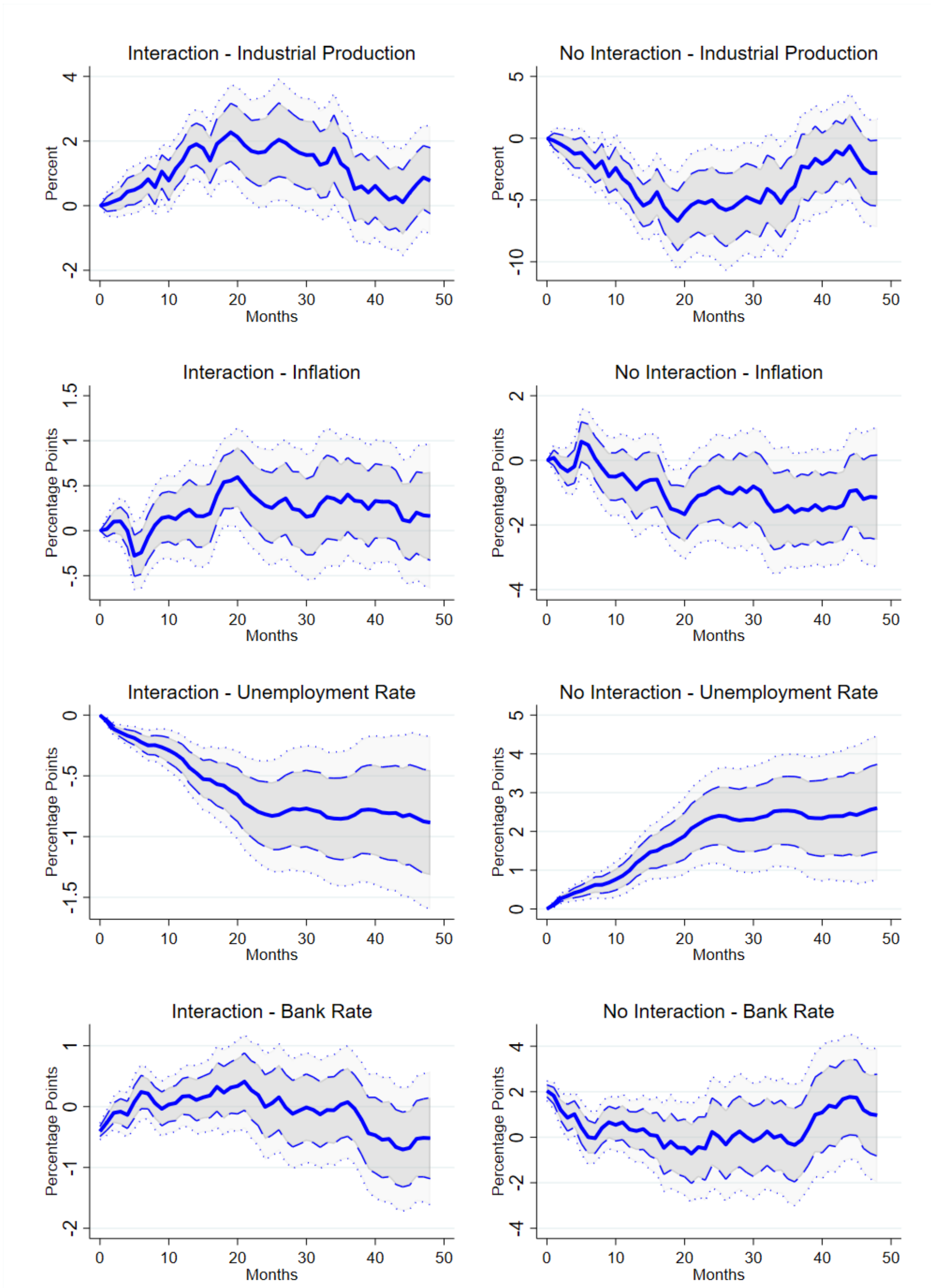
[59]Results without the recursiveness assumption, with face value, and with instrumental variables are also robust and are available upon request.

Figure F.3: local projection regressions with Macaulay duration for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of Macaulay duration. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Macaulay duration, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

Figure F.4: Local projection regressions with duration to GDP constructed from both nominal treasury bonds and inflation linked bonds for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed without the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and one lag of duration to GDP constructed with both nominally fixed rate bonds and inflation linked bonds. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with the Duration over GDP constructed with both nominally fixed rate bonds and inflation linked bonds, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

regime, in line with the US results and with the theoretical model.

G Model Derivations

G.1 Model Extended Derivations

In this appendix, I provide further derivations for theoretical model.

G.1.1 Government Debt

As a first illustrative step, Figure G.1 shows an example of the debt schedule following an issuance of one unit of debt on the various debt variables. Then, I show that the debt structure allows a parsimonious formulation for duration by providing the proof to Lemma 1.

Proof of Lemma 1.

Macaulay duration weights each cash flow of a debt instrument by its maturity and divides it by the net present value of these cash flows. In case of L_t new debt issued at prevailing new rate R_t^{new}:

$$Dur_t = \frac{\sum_{j=1}^{\infty} j \frac{(\delta^d + R_t^{new})(1-\delta^d)^{j-1}}{(1+R_t^{new})^j} L_t}{\sum_{j=1}^{\infty} \frac{(\delta^d + R_t^{new})(1-\delta^d)^{j-1}}{(1+R_t^{new})^j} L_t}$$

If we simplify and use the formula for geometric series we get,

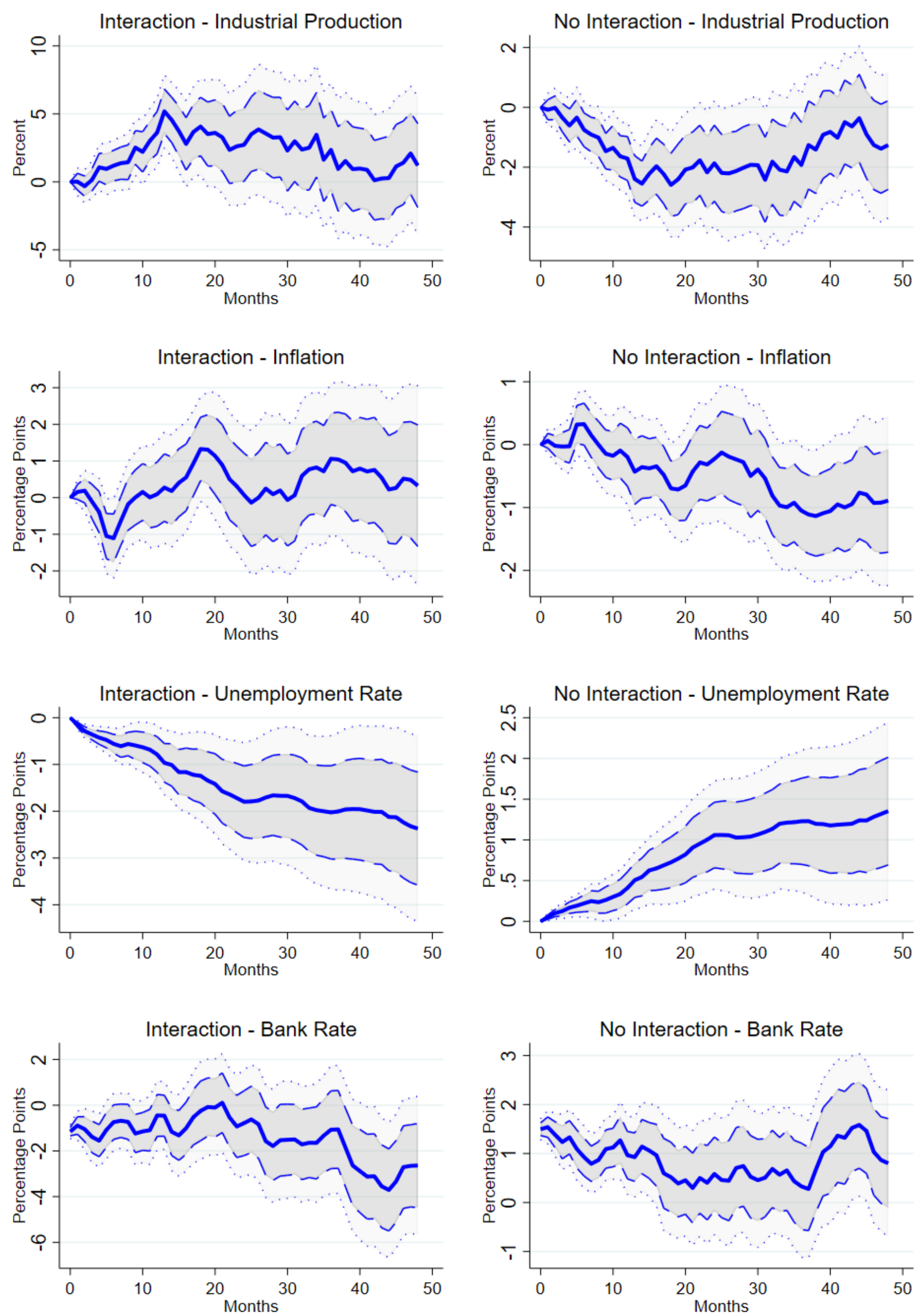
$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \sum_{j=1}^{\infty} j \left(\frac{1-\delta^d}{1+R_t^{new}} \right)^{j-1}$$

Take the sum from $j = 1$ to $j = 0$ due to the presence of j in the sum, recognize that we can express the expression inside the sum as a derivative,

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \sum_{j=0}^{\infty} j \left(\frac{1-\delta^d}{1+R_t^{new}} \right)^{j-1}$$

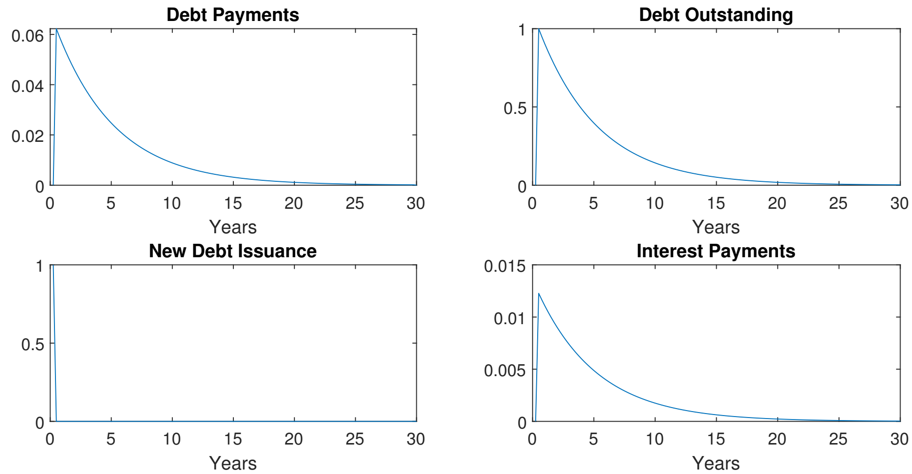
$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \sum_{j=0}^{\infty} \frac{d}{d\left(\frac{1-\delta^d}{1+R_t^{new}}\right)} \left(\frac{1-\delta^d}{1+R_t^{new}} \right)^{j}$$

Figure F.5: Smooth transition local projection interaction regressions for the UK



Notes: 68 and 90 confidence intervals. Newey-West standard errors. The sample goes from 1979m1 to 2007m12 with UK data. Identification of the monetary policy shock is achieved with the updated narrative method. Regressions performed with the recursiveness assumption on industrial production, Year on Year RPIX inflation, the commodity price index, and the unemployment rate. Each regression includes 48 lags of the monetary policy shock. In addition, each regression contains the first four lags of industrial production, Year on Year RPIX inflation, the commodity price index, the unemployment rate, and the Bank Rate and $F(DurGDP_{t-1})$, the smooth logistic transformation of the lag of duration to GDP. The speed of transition parameter θ is equal to 3. The specification is run in first differences for macroeconomic controls and in h-steps ahead difference for the left-hand-side variable. The first column shows the interaction term of the shock with $F(DurGDP_{t-1})$, the second column shows the the shock term not interacted. Each row shows a different LHS variable.

Figure G.1: Debt Repayment Schedule



Notes: these panels show the impact of an increase of one unit of new debt starting from no debt, that is $L_t = 1$ and $D_{t-1} = 0$, on debt dynamics. Time is quarterly, the interest rate on new debt R_t^{new} is 0.0123 (0.05 at annual frequency), the maturity parameter δ^d is equal to 0.05; therefore, duration is equal to 16.24 quarters. The first panel shows the overall debt payments in each future quarter F_{t+j}. The second panel shows the principal outstanding in each future quarter D_{t+j}. The third panel shows the flow of new issuances L_{t+j}, which is equal to 1 only in the first period. Finally, the fourth panel shows the interest payments $D_{t+j}R_{t+j}^{ave}$.

Use the formula for geometric series, retake the derivative, and simplify to obtain (7),

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \frac{d}{d\left(\frac{1-\delta^d}{1+R_t^{new}}\right)} \frac{1}{1 - \left(\frac{1-\delta^d}{1+R_t^{new}}\right)}$$

$$Dur_t = \frac{\delta^d + R_t^{new}}{1 + R_t^{new}} \frac{1}{\left(1 - \frac{1-\delta^d}{1+R_t^{new}}\right)^2}$$

$$Dur_t = \frac{1 + R_t^{new}}{\delta^d + R_t^{new}}$$

■

G.1.2 Households

The overall problem of the households

$$\max_{\{C_t, H_t, B_t^{crp}, B_t^{mp}, \{D_t^{t-j}\}_{j=0}^{\infty}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right]$$

$$s.t. \quad P_t C_t + B_t^{mp} + q_t^{crp} B_t^{crp} + \sum_{j=0}^{\infty} q_t^{t-j} D_t^{t-j} + P_t T_t = W_t H_t + P_t \Pi_t +$$

$$B_{t-1}^{mp}(1 + R_{t-1}^{mp}) + B_{t-1}^{crp}(1 + R_{t-1}^{crp}) \frac{P_t}{P_{t-1}} + \sum_{j=1}^{\infty} ((1-\delta^d) q_t^{t-j} + R_{t-j}^{new} + \delta^d) D_{t-1}^{t-j}$$

As a first step, I make the budget constraint real:

$$C_t + b_t^{mp} + q_t^{crp} b_t^{crp} + \sum_{j=0}^{\infty} q_t^{t-j} d_t^{t-j} + T_t = w_t H_t + \Pi_t +$$

$$b_{t-1}^{mp} \frac{1 + R_{t-1}^{mp}}{\pi_t} + b_{t-1}^{crp}(1 + R_{t-1}^{crp}) + \sum_{j=1}^{\infty} ((1-\delta^d) q_t^{t-j} + R_{t-j}^{new} + \delta^d) \frac{d_{t-1}^{t-j}}{\pi_t}$$

Write the Lagrangian and take the FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial C_t} &: \; C_t^{-\sigma} = \lambda_t \\
\frac{\partial L}{\partial H_t} &: \; \chi H_t^{\eta} = \lambda_t w_t \\
\frac{\partial L}{\partial b_t^{mp}} &: \; \lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{(1 + R_t^{mp})}{\pi_{t+1}} \right] \\
\frac{\partial L}{\partial b_t^{crp}} &: \; \lambda_t q_t^{crp} = \beta \mathbb{E}_t \left[\lambda_{t+1} (1 + R_t^{crp}) \right] \\
\frac{\partial L}{\partial d_t^{t-j}} &: \; \lambda_t q_t^{t-j} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \left((1-\delta^d) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta^d \right) \right]
\end{aligned}$$

We get a standard labor supply choice, a standard Euler for the monetary policy bond, the Euler for the corporate bond and the government bonds with the secondary market prices:

$$\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{\eta} \\
1 &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + R_t^{mp})}{\pi_{t+1}} \right] \\
1 &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1 + R_t^{crp})}{q_t^{crp}} \right] \\
q_t^{t-j} &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left((1-\delta^d) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta^d \right) \right]
\end{aligned}$$

The last condition is a standard asset pricing equation with the price of the bond today being equal to the capital gain on the non-matured portion of the bond $(1-\delta^d) q_{t+1}^{t-j}$ in addition to the payout being the promised fixed interest rate R_{t-j}^{new} and the repayment of the principal δ^d, all discounted by the SDF for nominal assets $\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}}$. As a next step, we substitute out the secondary market price with the friction Φ_t we can write the Euler equation for newly issued public debt in terms of tomorrows rate on new bonds as. Take the equation at $j = 0$:

$$q_t^t = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left((1-\delta^d) q_{t+1}^t + R_t^{new} + \delta^d \right) \right]$$

Expand the recursive term to be an infinite sum:

$$q_t^t = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left(R_t^{new} + \delta^d \right) \right] + \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} (1-\delta^d) q_{t+1}^t \right]$$

$$q_t^t = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\beta^j \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) (1-\delta^d)^{j-1} \left(R_t^{new} + \delta^d \right) \right]$$

Take to the left side terms at time t:

$$C_t^{-\sigma} \frac{q_t^t}{R_t^{new} + \delta^d} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\beta^j \left(C_{t+j} \right)^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) (1-\delta^d)^{j-1} \right]$$

Expand the first term and notice that the expression admits a recursive representation in term of the bonds issued in the following period.

$$\begin{aligned} C_t^{-\sigma} \frac{q_t^t}{R_t^{new} + \delta^d} &= \mathbb{E}_t \Bigg[\beta \left(C_{t+1} \right)^{-\sigma} \frac{1}{\pi_{t+1}} + \\ &\quad + (1-\delta^d) \beta \frac{1}{\pi_{t+1}} \sum_{j=1}^{\infty} \beta^j \left(C_{t+1+j} \right)^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+1+k}} \right) (1-\delta^d)^{j-1} \Bigg] \\ C_t^{-\sigma} \frac{q_t^t}{R_t^{new} + \delta^d} &= \mathbb{E}_t \left[\beta \left(C_{t+1} \right)^{-\sigma} \frac{1}{\pi_{t+1}} + (1-\delta^d) \beta \frac{1}{\pi_{t+1}} C_{t+1}^{-\sigma} \frac{q_{t+1}^{t+1}}{R_{t+1}^{new} + \delta^d} \right] \end{aligned}$$

Rearrange, substitute out the monetary policy rate Euler equation, and the primary market friction

$$\begin{aligned} \frac{q_t^t}{R_t^{new} + \delta^d} &= (1 + R_t^{mp})^{-1} + \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} (1-\delta^d) \frac{q_{t+1}^{t+1}}{R_{t+1}^{new} + \delta^d} \right] \\ \frac{1 + \Phi_t}{R_t^{new} + \delta^d} &= (1 + R_t^{mp})^{-1} + \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} (1-\delta^d) \frac{1 + \Phi_{t+1}}{R_{t+1}^{new} + \delta^d} \right] \end{aligned}$$

We do not need to carry around the prices of all other government bonds, but only of the currently issued one and the expected price on government bonds issued tomorrow. The interest rate on newly issued bonds today R_t^{new} depends on the current primary market frictions, on current monetary policy rates, and on the expected bond interest rates tomorrow and on tomorrows frictions in the primary market. Furthermore, the higher maturity is the higher the weight of futures rates compared to current short ones in determining the rate on newly issued government bonds. Furthermore, we can see that the Euler for public debt can be rewritten to equate the convolution of primary market friction and interest rates on newly issued bonds as a decaying average of a nominal yield curve of zero-coupon bonds:

$$\lambda_t (1 + \Phi_t) = \mathbb{E}_t \left[(\delta^d + R_t^{new}) \sum_{j=1}^{\infty} (1 - \delta^d)^{j-1} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) \beta^j \lambda_{t+j} \right]$$

$$\frac{(1 + \Phi_t)}{(\delta^d + R_t^{new})} = \sum_{j=1}^{\infty} (1 - \delta^d)^{j-1} \left[1 + R_t^{zerocoupon,t,t+j} \right]^{-j}$$

G.1.3 Public Debt Pricing

Proof of Lemma 2.

This subsection presents the derivation for the secondary market value of public debt. To this aim, we price separately each vintage of government bonds and then aggregate the price to the whole stock of debt. Take the Euler equation for a generic bond issued j periods ago:

$$q_t^{t-j} = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \left((1 - \delta^d) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta^d \right) \right]$$

Notice that we can proceed as in the $j = 0$ case and find:

$$C_t^{-\sigma} \frac{q_t^{t-j}}{R_{t-j}^{new} + \delta^d} = \mathbb{E}_t \sum_{j=1}^{\infty} \left[\beta^j (C_{t+j})^{-\sigma} \prod_{k=1}^{j} \left(\frac{1}{\pi_{t+k}} \right) (1 - \delta^d)^{j-1} \right]$$

This expression on the right is the same for all j as it is for $j = 0$, therefore we can equate them and write:

$$q_t^{t-j} = \frac{R_{t-j}^{new} + \delta^d}{R_t^{new} + \delta^d} (1 + \Phi_t)$$

Without the friction the price of a bond issued j periods ago is simply the ratio of the cashflows per period per unit of bond, with a higher interest rate legacy bond commanding a higher price. Notice also that the difference in interest rate has a higher impact in the case of longer maturity bonds (lower δ^d). As in the case of bonds issued in the current period and purchased on the secondary market, the price is higher with a higher primary market friction due to no arbitrage: if purchasing bonds on the primary market is more expensive their return will be higher to compensate, but this would push the price upward in the frictionless secondary market. Finally, we can define the secondary market value of public debt with an average price

q_t^d:

$$D_t q_t^d = \sum_{j=0}^{\infty}(1-\delta^d)^j L_{t-j} q_t^{t-j}$$

$$D_t q_t^d = \sum_{j=0}^{\infty}(1-\delta^d)^j L_{t-j} \frac{(\delta^d + R_{t-j}^{new})}{(\delta^d + R_t^{new})}(1+\Phi_t)$$

$$q_t^d = \frac{(\delta^d + R_t^{ave})}{(\delta^d + R_t^{new})}(1+\Phi_t)$$

Which is the same result as for each single bond, except with the average interest rate, this is due to the convenient geometric repayment formula. Notice that the price of government debt would increase following an increase in the primary market friction when we hold interest rates constant. When there is an increase in the primary market friction, the interest rate on newly issued bonds would increase, and depending on the shock and the maturity of the bond the impact can be higher than the friction itself and the value of public debt would decline. This is the case in my calibration following a monetary policy shock the price of debt and the primary market friction move in opposite directions when debt is long, but in the same one when debt is one period (mechanically as there $R_t^{ave} = R_t^{new}$).

$\blacksquare$

G.1.4 Duration and Debt Servicing Costs

Macaulay duration measures how much the value of debt changes following a one percent increase in interest rates across the yield curve. In this section, I prove Proposition 1 and I show how this is also a measure of how much insurance long debt provides against interest rate risk in terms of debt servicing costs. Specifically, I show how, following a one percent permanent increase in interest rates across the yield curve, duration measures the net present value of interest rate savings with long debt relative to short debt on existing debt.

Proof of Proposition 1.

Take the public debt structure presented in the model, with geometrically decaying principal. At period $t-1$ we have a stock of debt D_{t-1} with average interest rate R_{t-1}^{ave}. We will focus on changes on interest payments on existing debt, not on the whole stock of debt in equilibrium. For this reason, we consider new issuances to roll-over the existing stock of debt $L^{to\ roll-over_t}$ which is simply equal to the maturing fraction of the principal $\delta^d D_{t-1}$. From the law of motion of the principal of public debt we can see how this keeps this stock constant at D_{t-1}. Following an interest rate change to R_t^{new} we learn about at the end of period $t-1$, we can see how interest payments on rolled over legacy debt will simply be a weighted average of the new interest rate and the legacy one. This holds even if we go further in the future $j \geq 0$

as the change in R_t^{new} is permanent. The weight on legacy rates declines as we go further in the future as this debt is slowly maturing.

$$
\begin{aligned}
R_t^{ave,legacy} &= R_t^{new}\delta^d + R_{t-1}^{ave}\left(1-\delta^d\right)\\
R_{t+j}^{ave,legacy} &= R_t^{new}\delta^d \sum_{k=0}^{j}\left(1-\delta^d\right)^j + R_{t-1}^{ave}\left(1-\delta^d\right)^{j+1}\\
R_{t+j}^{ave,legacy} &= R_t^{new}\left[1-\left(1-\delta^d\right)^{j+1}\right] + R_{t-1}^{ave}\left(1-\delta^d\right)^{j+1}
\end{aligned}
$$

We discount the difference in interest payments between a short debt ($\delta^d = 1$),profile so that $R_{t+j}^{ave,legacy}|_{\delta^d=1} = R_t^{new}$ for $j \geq 0$ and the observed maturity with $\delta^d \leq 1$.

$$
\begin{aligned}
\sum_{j=0}^{\infty}\frac{R_t^{new}D_{t-1} - R_{t+j}^{ave,legacy}D_{t-1}}{(1+R_t^{new})^{j+1}} &= \sum_{j=0}^{\infty}\left(\frac{1-\delta^d}{1+R_t^{new}}\right)^{j+1} D_{t-1}(R_t^{new}-R_{t-1}^{ave})\\
&= \left(\frac{1+R_t^{new}}{\delta^d+R_t^{new}}-1\right)D_{t-1}(R_t^{new}-R_{t-1}^{ave})\\
&= \left(Dur_t - 1\right)D_{t-1}(R_t^{new}-R_{t-1}^{ave})
\end{aligned}
$$

We can see how the change in debt servicing costs following an interest rate change of $(R_t^{new} - R_{t-1}^{ave})$ is exactly equal to the difference in duration, where I used the result from Lemma 1, between debt with maturity parameter δ^d and debt with one period duration. This implies that the net present value of interest rate savings coming from long debt is correctly captured by its duration. Moreover, this maps as well to the changes in the market value of public debt. We take $\frac{q_t^d}{1+\Phi_t}$ as the overall market value (or the market value that accounts for the primary market friction) of legacy public debt from Lemma 2 and notice the same pattern:

$$
\begin{aligned}
\frac{q_t^d}{1+\Phi_t}D_{t-1} &= \frac{(\delta^d + R_t^{ave,legacy})}{(\delta^d+R_t^{new})}D_{t-1}\\
&= \frac{(\delta^d + R_t^{new}\delta^d + R_{t-1}^{ave}\left(1-\delta^d\right))}{(\delta^d+R_t^{new})}D_{t-1}\\
&= \left(1-\left(\frac{1+R_t^{new}}{\delta^d+R_t^{new}}-1\right)(R_t^{new}-R_{t-1}^{ave})\right)D_{t-1}\\
&= D_{t-1} - \left(Dur_t - 1\right)D_{t-1}(R_t^{new}-R_{t-1}^{ave})
\end{aligned}
$$

Following the same rate increase, the market value of public debt decreases exactly by the difference in duration between debt with maturity parameter δ^d and debt with one period duration. If we divide both terms nominal GDP ($Y_{t-1}^n \equiv P_{t-1}Y_{t-1}$) we obtain duration-to-GDP of legacy debt ($Dur_t D_{t-1}/Y_{t-1}^n$). This terminates the proof and establishes how we can use debt duration to measure how much insurance against interest rate changes the maturity profile gives both in term in debt servicing costs and valuation effects of the market value of public debt. ■

G.2 Calvo Retailers

In this section, I present a standard retailers problem to obtain the non linear New-Keynesian Phillips Curve. Retailers buy a wholesale good at price P_t^w and use it to produce the retail variety y_{it} with a linear technology that maps one to one the wholesale good to the retail variety. As each variety is differentiated they have market power and face a Calvo friction to change prices. Their real marginal cost $\mathcal{S}_t = \frac{P_t^w}{P_t} = \frac{1}{X_t}$ is the real wholesale price. The probability of not being able to reset prices is equal to θ in each period. The discounted present value of profits:

$$\mathbb{E}_t \sum_{j=0}^{\infty} SDF_{t,t+j} \left(P_{i,t+j} Y_{i,t+j} - P_{t+j} \mathcal{S}_{t+j} Y_{i,t+j} \right)$$

By looking at firms which cannot reset in the future, substituting the demand equation, taking first order conditions, and a little algebra, we can arrive to the standard non-linear New-Keynesian Phillips Curve[60]:

$$\begin{aligned}
K_t^f &= C_t^{-\sigma} Y_t \frac{1}{X_t} \frac{\varepsilon}{\varepsilon - 1} + \theta \beta \mathbb{E}_t \pi_{t+1}^{\varepsilon} K_{t+1}^f \\
F_t^f &= C_t^{-\sigma} Y_t + \theta \beta \mathbb{E}_t \pi_{t+1}^{\varepsilon - 1} F_{t+1}^f \\
\frac{K_t^f}{F_t^f} &= \left(\frac{1 - \theta \pi_t^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}}
\end{aligned}$$

G.3 Wholesalers

Wholesalers are perfectly competitive and they combine capital K_{t-1}, household labor H_t, and entrepreneurs labor H_t^e to make the wholesale goods:

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)}$$

They sell these goods at nominal price P_t^w to retailers. They pay nominal wage W_t for each unit of household labor, nominal W_t^e for each unit of entrepreneur labor, and real risky return R_t^r to capital owners. The capital share in production is α and Ω is the share of household labor in the overall labor employed by the firm. Technology is stochastic and its process is

[60] The full derivations are available upon request.

follows an AR(1) in logs:

$$\frac{A_t}{A} = \left(\frac{A_{t-1}}{A}\right)^{\rho^A} \exp(\varepsilon_t^A)$$

Their optimization problem is:

$$\max_{\{K_{t-1},H_t,H_t^e\}_{t=0}^{\infty}} \frac{1}{X_t} Y_t - W_t H_t - W_t^e H_t^e - R_t^t K_{t-1}$$

The solution to their optimization problem is:

$$\begin{aligned}
\frac{1}{X_t} \alpha \frac{Y_t}{K_{t-1}} &= R_t^r \\
\frac{1}{X_t} (1-\alpha)\Omega \frac{Y_t}{H_t} &= w_t \\
\frac{1}{X_t} (1-\alpha)(1-\Omega) \frac{Y_t}{H_t^e} &= w_t^e
\end{aligned}$$

G.4 Capital Producers

Capital producers are separate from entrepreneurs and combine investment resources I_t and legacy undepreciated capital $(1-\delta)K_{t-1}$ they purchase from entrepreneurs in order to sell new capital goods with objective function:

$$\max_{\{K_t,I_t,K_{t-1}\}_{t=0}^{\infty}} Q_t K_t - I_t - Q_t^{old}(1-\delta)K_{t-1}$$

These producers face a production with capital adjustment costs:

$$K_t = I_t - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + (1-\delta)K_{t-1}$$

By substituting the law of motion/production function and taking the first order conditions we can see the solution to the capital producers problem:

$$Q_t \left(1 - \phi_K \left(\frac{I_t}{K_{t-1}} - \delta\right)\right) = 1 \tag{G.1}$$

$$\frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}}\right)^2 - \frac{\phi_K}{2}\delta^2 = \frac{Q_t^{old} - Q_t}{Q_t}(1-\delta) \tag{G.2}$$

Equation (G.1) is a standard equation for Tobin's real capital price Q_t. With respect to the price of legacy capital Q_t^{old} priced by equation (G.2), I make the same simplification as BGG. For small perturbations close to the steady state, the price of the legacy capital stock and the newly produced one are the same.

G.5 Entrepreneurs

This section provides the detailed entrepreneurs problem solution. The first step is to show how to arrive to the participation constraint for the lenders. As discussed in the main text, an entrepreneur pays back debt if the return on her investment $(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1}$ is higher than the cost of servicing debt $Z^j_{t+1} B^j_t$ otherwise defaults and the lender recovers $(1-\mu)(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1}$. There exists a threshold $\bar{\omega}^j_{t+1}$ above which the entrepreneur pays a fixed amount and below the recovery value. From the perspective of the lender, the return on a single loan is therefore:

$$(1 + R^j_{t+1})B^j_t = \begin{cases} Z^j_{t+1} B^j_t & if\ \omega^j_{t+1} \geq \bar{\omega}^j_{t+1} \\ (1-\mu)(1 + R^k_{t+1})Q_t K_t \omega^j_{t+1} & if\ \omega^j_{t+1} < \bar{\omega}^j_{t+1} \end{cases}$$

The return on this loan in expected term (with respect to the idiosyncratic shock) but given a realized return R^k_{t+1}, must be equal to the outside option of lenders R^{crp}_t. This loan return is guaranteed for lenders as there is a large mass of entrepreneurs. We can take the expectation with respect to ω:

$$\begin{aligned}(1 + R^{crp}_t)B^j_t &= \int_0^{\bar{\omega}^j_{t+1}} (1-\mu)(1 + R^k_{t+1})Q_t K^j_t \omega f(\omega, \sigma_{\omega,t}) d\omega + \\ &+ \int_{\bar{\omega}^j_{t+1}}^{\infty} \bar{\omega}^j_{t+1}(1 + R^k_{t+1})Q_t K^j_t f(\omega, \sigma_{\omega,t}) d\omega \\ (1 + R^{crp}_t)(\kappa^j_t - 1) &= (1 + R^k_{t+1})\kappa^j_t(\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))\end{aligned}$$

Where, as in the main text, we define leverage as $\kappa^j_t \equiv \frac{Q_t K^j_t}{N^j_t}$, and the helping functions $\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) = \int_0^{\bar{\omega}^j_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega + \bar{\omega}^j_{t+1}(1 - F(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))$, $G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) = \int_0^{\bar{\omega}^j_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega$, and $F(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) = \int_0^{\bar{\omega}^j_{t+1}} f(\omega, \sigma_{\omega,t}) d\omega$. The entrepreneurs problem is to maximize expected wealth (so that the objective is linear), where they are protected by limited liability (so that the objective has the max operator), subject to the participation constraint of the lenders. They choose a combination of leverage κ^j_t before uncertainty is realized and default cut-off $\bar{\omega}^j_{t+1}$ contingent on shock realization:

$$\max_{\{\kappa^j_t, \bar{\omega}^j_{t+1}\}} \mathbb{E}_t \max\left[(1 + R^k_{t+1})\kappa^j_t N^j_t(\omega^j_{t+1} - \bar{\omega}^j_{t+1}), 0\right]$$

$$s.t.$$

$$(1 + R^{crp}_t)(\kappa^j_t - 1) = (1 + R^k_{t+1})\kappa^j_t(\Gamma(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}^j_{t+1}, \sigma_{\omega,t}))$$

This equivalent to:

$$\max_{\{\kappa_t^j,\bar{\omega}_{t+1}^j\}} L = \mathbb{E}_t\Bigg[\frac{(1+R_{t+1}^k)}{(1+R_t^{crp})}\kappa_t^j\left(\int_{\bar{\omega}_{t+1}^j}^{\infty}\omega f(\omega,\sigma_{\omega,t})d\omega - \bar{\omega}_{t+1}^j(1-F(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\right)+$$
$$-\lambda_{t+1}\left[(\kappa_t^j-1)-\frac{(1+R_{t+1}^k)}{(1+R_t^{crp})}\kappa_t^j(\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\right]\Bigg]$$

We can define the risk spread as the ratio of returns, $(1+s_{t+1}) \equiv \frac{(1+R_{t+1}^k)}{(1+R_t^{crp})}$, and notice that the objective function can be rewritten by taking advantage of the fact that the expected value of ω_{t+1} is 1: $1-\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}) = 1-\int_0^{\bar{\omega}_{t+1}^j}\omega f(\omega,\sigma_{\omega,t})d\omega - \bar{\omega}_{t+1}^j(1-F(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})) = \int_0^{\bar{\omega}_{t+1}^j}\omega f(\omega,\sigma_{\omega,t})d\omega + \int_{\bar{\omega}_{t+1}^j}^{\infty}\omega f(\omega,\sigma_{\omega,t})d\omega - \int_0^{\bar{\omega}_{t+1}^j}\omega f(\omega,\sigma_{\omega,t})d\omega - \bar{\omega}_{t+1}^j(1-F(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})) = \int_{\bar{\omega}_{t+1}^j}^{\infty}\omega f(\omega,\sigma_{\omega,t})d\omega - \bar{\omega}_{t+1}^j(1-F(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))$:

$$\max_{\{\kappa_t^j,\bar{\omega}_{t+1}^j\}} L = \mathbb{E}_t\Bigg[(1+s_{t+1})\kappa_t^j\left(1-\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})\right)+$$
$$-\lambda_{t+1}\left[1-(1+s_{t+1})\frac{\kappa_t^j}{\kappa_t^j-1}(\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\right]\Bigg]$$

We can solve this problem by taking first order conditions:

$$\frac{\partial L}{\partial \kappa_t^j}:\ \mathbb{E}_t\Bigg[(1+s_{t+1})\left(1-\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})\right)$$
$$-\lambda_{t+1}(1+s_{t+1})\frac{1}{(\kappa_t^j-1)^2}(\Gamma(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\Bigg]=0$$
$$\frac{\partial L}{\partial \bar{\omega}_{t+1}^j}:\ -(1+s_{t+1})\kappa_t^j\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})$$
$$+\lambda_{t+1}\left[(1+s_{t+1})\frac{\kappa_t^j}{\kappa_t^j-1}(\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\right]=0$$

To solve the problem, first simplify the first order condition with respect to the threshold:

$$-\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})+\lambda_{t+1}\left[\frac{1}{\kappa_t^j-1}(\Gamma_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t})-\mu G_\omega(\bar{\omega}_{t+1}^j,\sigma_{\omega,t}))\right]=0$$

This equation defines the Lagrange multiplier λ_{t+1} in terms of $\bar{\omega}_{t+1}^j$ and, once we substitute this in the first first order condition we have a $\bar{\omega}_{t+1}^j$ in terms of the ratio $(1+s_{t+1})$. We can use the participation constraint to find the mapping from the ratio $(1+s_{t+1})$ to leverage κ_t^j. As the first order conditions are the same for all entrepreneurs irrespective of their equity level, the leverage and threshold choices are the same for all: κ_t and $\bar{\omega}_{t+1}$. To ease notation let

$\Gamma_{\omega,t+1} \equiv \Gamma_\omega(\bar{\omega}^j_{t+1}, \sigma_{\omega,t})$ and similarly for G and other derivatives.

$$
\begin{aligned}
\lambda_{t+1} &= \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}}(\kappa_t - 1) \\
0 &= \mathbb{E}_t \left[(1+s_{t+1})\left(1-\Gamma_{t+1}\right) - \lambda_{t+1}(1+s_{t+1})\frac{1}{(\kappa_t-1)^2}(\Gamma_{t+1} - \mu G_{t+1}) \right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1} - \mu G_{t+1})
\end{aligned}
$$

Plug the Lagrangian multiplier in the second equation to reduce the system to 2 equations:

$$
\begin{aligned}
0 &= \mathbb{E}_t \left[(1+s_{t+1})\left(1-\Gamma_{t+1}\right) - \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}}(1+s_{t+1})\frac{1}{(\kappa_t-1)}(\Gamma_{t+1} - \mu G_{t+1}) \right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1} - \mu G_{t+1})
\end{aligned}
$$

Use participation constraint to simplify the first equation. These two equations summarize the non-linear system for the entrepreneur choice:

$$
\begin{aligned}
0 &= \mathbb{E}_t \left[(1+s_{t+1})\kappa_t\left(1-\Gamma_{t+1}\right) - \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}} \right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1+s_{t+1})(\Gamma_{t+1} - \mu G_{t+1})
\end{aligned}
$$

To substitute out the threshold choice we have to log-linearize the system as $\bar{\omega}_{t+1}$ is now defined implicitly, as done in the linearization section. To close the description of the entrepreneur sector we need to specify how the wealth and consumption of entrepreneurs behave. As a first step, I show the value of equity today is the return on capital invested less the amount paid to the lender from last period:

$$
\begin{aligned}
V_t &= (1+R^k_t)Q_{t-1}K_{t-1} \\
&\quad - \left((1+R^{crp}_{t-1}) + \frac{\mu(1+R^k_t)Q_{t-1}K_{t-1}}{Q_{t-1}K_{t-1} - N_{t-1}} \int_0^{\bar{\omega}_t} \omega dF(\omega, \sigma_{\omega,t-1}) \right) (Q_{t-1}K_{t-1} - N_{t-1})
\end{aligned}
$$

We can expand this to describe it in terms of equity amount in the previous period and leverage:

$$
\begin{aligned}
V_t &= \left((1+R^k_t)\kappa_{t-1} - \left((1+R^{crp}_{t-1}) + \mu\frac{(1+R^k_t)\kappa_{t-1}}{\kappa_{t-1}-1} G(\bar{\omega}_t, \sigma_{\omega,t-1}) \right) (\kappa_{t-1} - 1) \right) N_{t-1} \\
V_t &= \left((R^k_t - R^{crp}_{t-1})\kappa_{t-1} + (1+R^{crp}_{t-1}) - \mu(1+R^k_t)\kappa_{t-1} G(\bar{\omega}_t, \sigma_{\omega,t-1}) \right) N_{t-1}
\end{aligned}
$$

Based on this result we can define the return on equity as what maps the equity quantity last period into the equity value today:

$$(1+R_t^e) = \big((R_t^k - R_{t-1}^{crp})\kappa_{t-1} + (1+R_{t-1}^{crp}) - \mu(1+R_t^k)\kappa_{t-1}G(\bar{\omega}_t, \sigma_{\omega,t-1})\big)$$

From there the last two equations presented in the main text follow directly. The new equity is equal to the return on last period equity for the entrepreneurs who do not exit in addition to labor income. Entrepreneurs who exit consume the value of the firm:

$$\begin{aligned}
N_t &= \gamma(1+R_t^e)N_{t-1} + w_t^e \\
C_t^e &= (1-\gamma)(1+R_t^e)N_{t-1}
\end{aligned}$$

G.6 All Equilibrium Condition

The competitive equilibrium consists of 16 endogenous allocations $\{C_t,\ C_t^e,\ I_t,\ Y_t,\ K_t^f,\ F_t^f, \kappa_t,\ \bar{\omega}_t,\ N_t,\ K_t,\ H_t,\ H_t^e,\ l_t,\ f_t,\ d_t,\ T_t\}$, 15 prices $\{w_t,\ w_t^e,\ \pi_t,\ R_t,\ R_t^{crp},\ R_t^k,\ R_t^r,\ R_t^e,\ s_t,\ X_t,\ Q_t,\ \Phi_t,\ R_t^{new},\ R_t^{ave},\ R_t^{mp}\}$, and 4 exogenous processes $\{G_t,\ \sigma_{\omega,t},\ A_t,\ \nu_t^{\Phi}\}$; such that households, primary market participants, final good producers, retailers, wholesalers, capital producers, and entrepreneurs optimize, the central bank follows a Taylore rule, the treasury follows the tax rule, and markets clear. The equilibrium is characterized by the following equations and processes:

$$\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{\eta} \\
C_t^{-\sigma} &= \mathbb{E}_t\left[\beta C_{t+1}^{-\sigma}\frac{(1+R_t^{mp})}{\pi_{t+1}}\right] \\
K_t^f &= C_t^{-\sigma} Y_t \frac{1}{X_t}\frac{\varepsilon}{\varepsilon-1} + \theta\beta\mathbb{E}_t\pi_{t+1}^{\varepsilon}K_{t+1}^f \\
F_t^f &= C_t^{-\sigma} Y_t + \theta\beta\mathbb{E}_t\pi_{t+1}^{\varepsilon-1}F_{t+1}^f \\
\frac{K_t^f}{F_t^f} &= \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} \\
Y_t &= A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}(H_t^e)^{(1-\alpha)(1-\Omega)} \\
R_t^r &= \frac{1}{X_t}\alpha\frac{Y_t}{K_{t-1}} \\
w_t &= \frac{1}{X_t}(1-\alpha)\Omega\frac{Y_t}{H_t} \\
w_t^e &= \frac{1}{X_t}(1-\alpha)(1-\Omega)\frac{Y_t}{H_t^e}
\end{aligned}$$

$$K_t = I_t - \frac{\phi_K}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + (1-\delta)K_{t-1}$$
$$Q_t\left(1 - \phi_K\left(\frac{I_t}{K_{t-1}} - \delta\right)\right) = 1$$
$$1 + R_{t+1}^k = \frac{\frac{1}{X_{t+1}}\alpha\frac{Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}$$

$$\begin{aligned}
\kappa_t &= \frac{Q_t K_t}{N_t} \\
1 + s_t &= \frac{1 + R_t^k}{1 + R_{t-1}^{crp}} \\
0 &= \mathbb{E}_t\left[(1 + s_{t+1})\kappa_t\left(1 - \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t})\right) - \frac{\Gamma_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\Gamma_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - \mu G_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t})}\right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1 + s_{t+1})(\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - \mu G(\bar{\omega}_{t+1}, \sigma_{\omega,t})) \\
N_t &= \gamma(1 + R_t^e)N_{t-1} + w_t^e \\
C_t^e &= (1-\gamma)(1 + R_t^e)N_{t-1} \\
1 + R_t^e &= \left((R_t^k - R_{t-1}^{crp})\kappa_{t-1} + (1 + R_{t-1}^{crp}) - \mu(1 + R_t^k)\kappa_{t-1}G(\bar{\omega}_t, \sigma_{\omega,t-1})\right) \\
Y_t &= C_t + C_t^e + I_t + G_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1})(1 + R_t^k)N_{t-1}\kappa_{t-1} \\
H_t^e &= 1 \\
f_t &= (R_{t-1}^{ave} + \delta^d)\frac{1}{\pi_t}d_{t-1} \\
d_t &= (1 - \delta^d)\frac{1}{\pi_t}d_{t-1} + l_t \\
R_t^{ave} &= \left(1 - \frac{l_t}{d_t}\right)R_{t-1}^{ave} + \frac{l_t}{d_t}R_t^{new} \\
f_t &= T_t - G_t + l_t \\
T_t &= G_t + (T - G)\left(\frac{d_{t-1}}{d}\right)^{\tau_T} \\
\hat{\Phi}_t &= \zeta(\hat{b}_t^{crp} + \hat{l}_t) + \nu_t^\Phi \\
\frac{(1 + \Phi_t)}{(\delta^d + R_t^{new})} &= (1 + R_t^{mp})^{-1} + \mathbb{E}_t\left[\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}(\pi_{t+1})}(1 - \delta^d)\frac{(1 + \Phi_{t+1})}{(\delta^d + R_{t+1}^{new})}\right] \\
C_t^{-\sigma} &= \mathbb{E}_t\left[\beta C_{t+1}^{-\sigma}\frac{(1 + R_t^{crp})}{1 + \Phi_t}\right] \\
1 + R_t &= \frac{(1 + R_t^{crp})}{1 + \Phi_t}
\end{aligned}$$

$$
\begin{aligned}
\left(\frac{1+R_t^{mp}}{1+R^{mp}}\right)^{1+R^{mp}} &= \left(\frac{1+R_{t-1}^{mp}}{1+R^{mp}}\right)^{\rho^{mp}(1+R^{mp})} \left[\left(\frac{\mathbb{E}_t\pi_{t+1}}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right]^{(1-\rho^{mp})} \exp(\varepsilon_t^{mp}) \\
\frac{G_t}{G} &= \left(\frac{G_{t-1}}{G}\right)^{\rho^g} \exp(\varepsilon_t^g) \\
\frac{A_t}{A} &= \left(\frac{\hat{A}_{t-1}}{A}\right)^{\rho^A} \exp(\varepsilon_t^A) \\
\frac{\sigma_{\omega,t}}{\sigma_\omega} &= \left(\frac{\sigma_{\omega,t-1}}{\sigma_\omega}\right)^{\rho^{\sigma_\omega}} \exp(\varepsilon_t^{\sigma_\omega}) \\
\nu_t^\Phi &= \rho^\Phi \nu_{t-1}^\Phi + \varepsilon_t^\Phi
\end{aligned}
$$

G.7 Steady State

In this section I derive the steady state around a zero inflation steady state. Steady state variables are written without the time subscript. We assume an exogenous level of government consumption to GDP $\bar{G} \equiv G/Y$. I follow BGG and, for the financial accelerator, I target the steady state level of the financial accelerator friction s and the leverage level κ, and the average default rate $F(\bar{\omega}, \sigma_\omega)$ and obtain the resulting default threshold $\bar{\omega}$, monitoring cost μ, and volatility of idiosyncratic productivity shocks σ_ω. I take a similar route for the primary marker friction and I target the steady state friction Φ and the impact of an increase in debt by one percent of GDP to rates ζ. I find the implied parameter of the financial accelerator by solving non-linearly:

$$
\begin{aligned}
0 &= \left[(1+s)\kappa\left(1-\Gamma(\bar{\omega}, \sigma_{\omega,t})\right) - \frac{\Gamma_\omega(\bar{\omega}, \sigma_\omega)}{\Gamma_\omega(\bar{\omega}, \sigma_\omega) - \mu G_\omega(\bar{\omega}, \sigma_{\omega,t})}\right] \\
\frac{\kappa-1}{\kappa} &= (1+s)(\Gamma(\bar{\omega}, \sigma_\omega) - \mu G(\bar{\omega}, \sigma_\omega)) \\
F(\bar{\omega}, \sigma_{\omega,t}) &= Default\ Rate
\end{aligned}
$$

With:

$$
\begin{aligned}
G(\bar{\omega}, \sigma_\omega) &= \Phi\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sigma_\omega}\right) \\
F(\bar{\omega}, \sigma_\omega) &= \Phi\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sigma_\omega}\right) \\
\Gamma(\bar{\omega}, \sigma_\omega) &= G(\bar{\omega}, \sigma_\omega) + \bar{\omega}(1 - F(\bar{\omega}, \sigma_\omega)) \\
G_\omega(\bar{\omega}, \sigma_\omega) &= \bar{\omega} f(\bar{\omega}, \sigma_\omega) \\
\Gamma_\omega(\bar{\omega}, \sigma_\omega) &= (1 - F(\bar{\omega}, \sigma_\omega))
\end{aligned}
$$

Notice that these conditions also restrict the value of the death rate of entrepreneurs γ as shown below. The price of capital and the investment rate:

$$
\begin{aligned}
I &= \delta K \\
Q &= 1
\end{aligned}
$$

Then solve the interest rates:

$$
\begin{aligned}
1 + R^n &= \frac{1}{\beta} \\
1 + R^{crp} &= (1 + R^n)(1 + \Phi) \\
1 + R^k &= (1 + R^{crp})(1 + s) \\
1 + R^r &= R^k - (1 + \delta)
\end{aligned}
$$

Find the steady state values of the Phillips curve variables and markup:

$$
\begin{aligned}
K^f &= \frac{1}{1 - \theta\beta} C^{-\sigma} Y \\
F^f &= \frac{1}{1 - \theta\beta} C^{-\sigma} Y \\
\frac{1}{X} &= \frac{\varepsilon}{\varepsilon - 1}
\end{aligned}
$$

Let's solve for the average return on invested entrepreneur wealth:

$$
1 + R^e = \big((R^k - R^{crp})\kappa + (1 + R^{crp}) - \mu(1 + R^k)\kappa G(\bar{\omega}, \sigma_\omega)\big)
$$

That can be simplified with steady states relations in terms of explicitly chosen values:

$$
\begin{aligned}
1 + R^e &= \big((R^k - R^{crp})\kappa + (1 + R^{crp}) - \mu(1 + R^k)\kappa G(\bar{\omega}, \sigma_\omega)\big) \\
1 + R^e &= (1 + R^{crp})(s + 1)\kappa \left(1 - \Gamma(\bar{\omega}, \sigma_\omega)\right)
\end{aligned}
$$

This allows to write entrepreneur wealth in term of the entrepreneur wage:

$$
N = \frac{1}{1 - \gamma(1 + R^e)} w^e
$$

Notice that, we can express the capital stock in terms of hours, net return on capital, and parameters from the first order condition of the firm with respect to capital:

$$
\begin{aligned}
R^r &= \alpha \frac{A}{X} K^{\alpha-1} H^{(1-\alpha)\Omega} \\
K &= \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{1}{1-\alpha}} H^{\Omega}
\end{aligned}
$$

From here we can also express entrepreneurs wage by substituting out capital:

$$
\begin{aligned}
w^e &= (1-\alpha)(1-\Omega) \frac{A}{X} K^{\alpha} H^{(1-\alpha)\Omega} \\
w^e &= (1-\alpha)(1-\Omega) \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{\alpha}{1-\alpha}} H^{\Omega}
\end{aligned}
$$

We can now solve for the implied death rate of entrepreneurs γ by equating the expression of entrepreneur wealth in term of the entrepreneur wage and the in term of capital and leverage:

$$
\begin{aligned}
N &= N \\
\frac{1}{1-\gamma(1+R^e)} w^e &= \frac{1}{\kappa} K \\
\frac{1}{1-\gamma(1+R^e)} (1-\alpha)(1-\Omega) \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{\alpha}{1-\alpha}} H^{\Omega} &= \frac{1}{\kappa} \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{1}{1-\alpha}} H^{\Omega} \\
\gamma &= \frac{1 - \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa}{(1+R^e)}
\end{aligned}
$$

From here we can set hours to one in steady state and find all resulting steady state values. Notice that this will imply a value for χ.

$$
\begin{aligned}
H &= 1 \\
K &= \left(\frac{A}{X}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{R^r}\right)^{\frac{1}{1-\alpha}} \\
N &= \frac{1}{\kappa} K \\
Y &= A K^{\alpha} \\
w^e &= (1-\alpha)(1-\Omega) \frac{Y}{X} \\
G &= \bar{G} Y
\end{aligned}
$$

$$
\begin{aligned}
I &= \delta K \\
C^e &= (1-\gamma)(1+R^e)N \\
C &= Y - G - I - C^e - \mu G(\bar{\omega}, \sigma_\omega)(1+R^k)\kappa N \\
\chi &= W H^{-\eta} C^{-\sigma}
\end{aligned}
$$

The long government bond Euler in steady state:

$$
\begin{aligned}
\frac{(1+\Phi)}{(\delta^d + R^{new})} &= (1+R^{mp})^{-1} + \left[\beta \frac{C^{-\sigma}}{C^{-\sigma}\pi}(1-\delta^d)\frac{(1+\Phi)}{(\delta^d + R^{new})}\right] \\
R^{new} &= R^{mp}(1+\Phi) + \delta^d \Phi
\end{aligned}
$$

We are targeting a level of public debt to GDP in steady state: $\bar{d} = \frac{d}{Y}$, this implies we can find the values for the government variables, start with new issuances:

$$
\begin{aligned}
l &= \delta^d d \\
l &= \delta^d \bar{d} Y
\end{aligned}
$$

Repayments:

$$
n = (R^{new} + \delta^d) d
$$

Now to find the tax in steady state combine these results with the government budget constraint:

$$
\begin{aligned}
n &= T - G + l \\
T &= (R^{new}\bar{d} + \bar{G})Y
\end{aligned}
$$

G.8 Log-linearization

I linearize all equilibrium conditions around a zero inflation steady state. Debt quantity variables are linearized over steady state GDP so that $\hat{D}_t = \frac{D_t - D}{Y}$ in order to interpret the results as changes in debt over GDP, I do this as the main economic channel of debt supply goes through a volume effect and a standard percent deviation would not capture it. Moreover, interest rate variables are linearized so that $\hat{R}^{crp}_t = R^{crp}_t - R^{crp}$ in order to interpret results as percentage point deviations, this includes the two spread friction variables Φ_t and s_t. Finally, I log-linearize all other variables so that $\hat{C}_t = \frac{C_t - C}{C}$[61].

I present the log-linearization of the entrepreneur problem, which could be of independent interest to the reader and of the primary market participants to illustrate the methodology.

[61] The MPK, R^r is log-linearized

With respect to the other sectors of the economy, I present the resulting linearized equations, the full derivations are available upon request.

G.8.1 Primary Dealers Financial Friction

The primary market friction:

$$\Phi_t = \Phi_0 \left(b_t^{crp} + l_t\right)^{\Phi_1} \exp\left(\frac{\nu_t^{\Phi}}{\Phi}\right)$$

In steady state the friction is:

$$\Phi = \Phi_0 \left(b^{crp} + l\right)^{\Phi_1}$$

As Φ_t represents a spread it is already in percentage points, therefore we linearize it instead of log-linearize it, moreover, for all debt variables we take deviations over gdp rather than over its steady state:

$$\begin{aligned}
\hat{\Phi}_t &= \Phi\Phi_1 \left(\frac{Y}{b^{crp} + l}\frac{db_t^{crp}}{Y} + \frac{Y}{b^{crp} + l}\frac{dl_t}{Y}\right) + \Phi\frac{1}{\Phi}\exp\left(\frac{0}{\Phi}\right)\nu_t^{\Phi} \\
\hat{\Phi}_t &= \Phi\Phi_1 \left(\frac{Y}{b^{crp} + l}\hat{b}_t^{crp} + \frac{Y}{b^{crp} + l}\hat{l}_t\right) + \nu_t^{\Phi} \\
\hat{\Phi}_t &= \zeta\left(\hat{b}_t^{crp} + \hat{l}_t\right) + \nu_t^{\Phi}
\end{aligned}$$

G.8.2 Entrepreneurs

Leverage - Spread Definitions. Start with definitions of leverage and spreads:

$$\begin{aligned}
\kappa_t &= \frac{Q_t K_t}{N_t} \\
\hat{\kappa}_t &= \hat{Q}_t + \hat{K}_t - \hat{N}_t \\
1 + s_t &= \frac{1 + R_t^k}{1 + R_{t-1}^{crp}} \\
\frac{1}{1+s}\hat{s}_t &= \frac{1}{1+R^k}\hat{R}_t^k - \frac{1}{1+R^{crp}}\hat{R}_{t-1}^{crp}
\end{aligned}$$

Leverage - Spread Relation. Now move to the two non-linear equilibrium conditions:

$$\begin{aligned}
0 &= \mathbb{E}_t\left[(1 + s_{t+1})\kappa_t\left(1 - \Gamma_{t+1}\right) - \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}}\right] \\
\frac{\kappa_t - 1}{\kappa_t} &= (1 + s_{t+1})(\Gamma_{t+1} - \mu G_{t+1})
\end{aligned}$$

To substitute out the threshold choice we have to log-linearize the system as $\bar{\omega}_{t+1}$ is now defined implicitly. Start with the participation constraint. To ease notation let variables without subscripts being steady state values $\Gamma_\omega \equiv \Gamma_\omega(\bar{\omega}_{ss}, \sigma_{\omega,ss})$ and similarly for G and other derivatives.

$$\frac{1}{\kappa}\hat{\kappa}_t = (1+s)(\Gamma - \mu G)\frac{\hat{s}_{t+1}}{1+s} + (1+s)(\Gamma_\omega - \mu G_\omega)\bar{\omega}\hat{\omega}_{t+1} + (1+s)(\Gamma_\sigma - \mu G_\sigma)\sigma_\omega\hat{\sigma}_{\omega,t}$$

Simplify with steady state relationship:

$$\frac{1}{\kappa - 1}\hat{\kappa}_t = \frac{\hat{s}_{t+1}}{1+s} + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}\bar{\omega}\hat{\omega}_{t+1} + \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega\hat{\sigma}_{\omega,t}$$

Make explicit $\bar{\omega}\hat{\omega}_{t+1}$:

$$\bar{\omega}\hat{\omega}_{t+1} = \frac{\Gamma - \mu G}{\Gamma_\omega - \mu G_\omega}\left[\frac{1}{\kappa - 1}\hat{\kappa}_t - \frac{\hat{s}_{t+1}}{1+s} - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

Now log-linearize the optimal choice:

$$\begin{aligned}0 = \mathbb{E}_t\Bigg[&(1+s)\kappa\left(1-\Gamma\right)\hat{\kappa}_t + (1+s)\kappa\left(1-\Gamma\right)\frac{\hat{s}_{t+1}}{1+s} - (1+s)\kappa\Gamma_\omega\hat{\omega}_{t+1} - (1+s)\kappa\Gamma_\sigma\hat{\sigma}_{\omega,t}\\ &+ \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{(\Gamma_\omega - \mu G_\omega)^2}\bar{\omega}\hat{\omega}_{t+1} + \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{(\Gamma_\omega - \mu G_\omega)^2}\sigma_\omega\hat{\sigma}_{\omega,t}\Bigg]\end{aligned}$$

Use steady state relationship:

$$\begin{aligned}0 = \mathbb{E}_t\Bigg[&\hat{\kappa}_t + \frac{\hat{s}_{t+1}}{1+s} - \frac{\Gamma_\omega}{(1-\Gamma)}\bar{\omega}\hat{\omega}_{t+1} - \frac{\Gamma_\sigma}{(1-\Gamma)}\sigma_\omega\hat{\sigma}_{\omega,t}\\ &+ \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\bar{\omega}\hat{\omega}_{t+1} + \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\sigma_\omega\hat{\sigma}_{\omega,t}\Bigg]\end{aligned}$$

Group variables:

$$\begin{aligned}0 = \mathbb{E}_t\Bigg[&\hat{\kappa}_t + \frac{\hat{s}_{t+1}}{1+s}\\ &+ \left[\mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)} - \frac{\Gamma_\omega}{(1-\Gamma)}\right]\bar{\omega}\hat{\omega}_{t+1} + \left[\mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)} - \frac{\Gamma_\sigma}{(1-\Gamma)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\Bigg]\end{aligned}$$

Make explicit $\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}$:

$$\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1} = \frac{1}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]}\left[\hat{\kappa}_t + \mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s} - \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu\frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

In this way we can eliminate $\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}$ by taking expectations of the log-linearized participation constraint and equating it with the equation we just derived:

$$\frac{\Gamma-\mu G}{\Gamma_\omega-\mu G_\omega}\left[\frac{1}{\kappa-1}\hat{\kappa}_t-\mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s}-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$
$$=\frac{1}{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]}\left[\hat{\kappa}_t+\mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s}-\left[\frac{\Gamma_\sigma}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\sigma}G_\omega-\Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

Some algebra:

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\left[\frac{1}{\kappa-1}\hat{\kappa}_t-\mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s}-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$
$$=\left[\hat{\kappa}_t+\mathbb{E}_t\frac{\hat{s}_{t+1}}{1+s}-\left[\frac{\Gamma_\sigma}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\sigma}G_\omega-\Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\sigma_\omega\hat{\sigma}_{\omega,t}\right]$$

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}+1=\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]+\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$$

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\frac{1}{\kappa-1}-1=\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\frac{1}{\kappa-1}-\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$$
$$=\frac{\left[\frac{\Gamma_\omega-\mu G_\omega}{\kappa s}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\frac{1}{\kappa-1}-\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$$
$$=\frac{\left[\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}(\kappa-1)-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\frac{1}{\kappa-1}-\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$$
$$=-\frac{\left[\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\frac{1}{\kappa-1}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$$

$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}-\left[\frac{\Gamma_\sigma}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\sigma}G_\omega-\Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]=$$
$$\frac{\left[\frac{\Gamma_\omega}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\omega}G_\omega-\Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}-\left[\frac{\Gamma_\sigma}{(1-\Gamma)}-\mu\frac{\Gamma_{\omega\sigma}G_\omega-\Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega-\mu G_\omega)}\right]\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$$

Use these results to make explicit the spread relationship with leverage and risk:

$$\frac{\mathbb{E}_t s_{t+1}}{1+s} = -\frac{\mu \frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu \frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} \frac{1}{\kappa - 1}\hat{\kappa}_t +$$
$$+ \frac{-\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu \frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G} + \left[\frac{\Gamma_\sigma}{(1-\Gamma)} - \mu \frac{\Gamma_{\omega\sigma}G_\omega - \Gamma_\omega G_{\omega\sigma}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\left[\frac{\Gamma_\omega}{(1-\Gamma)} - \mu \frac{\Gamma_{\omega\omega}G_\omega - \Gamma_\omega G_{\omega\omega}}{\Gamma_\omega(\Gamma_\omega - \mu G_\omega)}\right] + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} \sigma_\omega \hat{\sigma}_{\omega,t}$$

Law of Motion of N. To find the law of motion of entrepreneur's wealth we are going to use a number of steady state relationships:

$$1 + R^e = (1 + R^{crp})(1+s)\kappa\,(1 - \Gamma(\bar{\omega}, \sigma_\omega))$$
$$1 - \gamma(1 + R^e) = \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa$$

Start with the average return on entrepreneur's wealth:

$$1 + R^e_t = \left((R^k_t - R^{crp}_{t-1})\kappa_{t-1} + (1 + R^{crp}_{t-1}) - \mu(1 + R^k_t)\kappa_{t-1} G(\bar{\omega}_t, \sigma_{\omega,t-1})\right)$$
$$\hat{R}^e_t = \kappa\left(\hat{R}^k_t - \hat{R}^{crp}_{t-1}\right) + \kappa(1 + R^{crp})(s)\hat{\kappa}_{t-1} + \hat{R}^{crp}_{t-1}$$
$$- \mu G(\bar{\omega}, \sigma_\omega)(1 + R^{crp})(1+s)\kappa\left(\frac{\hat{R}^k_t}{1 + R^k} + \hat{\kappa}_{t-1} + \frac{G_\omega \bar{\omega}}{G(\bar{\omega}, \sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma \sigma_\omega}{G(\bar{\omega}, \sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right)$$

Notice that, as monitoring costs $\mu G(\bar{\omega}, \sigma_\omega)$ are small in the proposed calibration, the terms in the second line will negligible. Now move to the law of motion of entrepreneur's wealth:

$$N_t = \gamma(1 + R^e_t)N_{t-1} + w^e_t$$
$$N\hat{N}_t = \gamma(1 + R^e)N\left(\frac{\hat{R}^e_t}{1 + R^e} + \hat{N}_{t-1}\right) + N\left(1 - \gamma R^e\right)\hat{w}^e_t$$
$$\hat{N}_t = \gamma(1 + R^e)\left(\frac{\hat{R}^e_t}{1 + R^e} + \hat{N}_{t-1}\right) + \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa \hat{w}^e_t$$

Entrepreneur's Consumption. Finally, let's log-linarize the entrepreneur's consumption in a similar way as for the law of motion of N:

$$C^e_t = (1-\gamma)(1 + R^e_t)N_{t-1}$$
$$\hat{C}^e_t = \hat{N}_{t-1} + \frac{\hat{R}^e_t}{1 + R^e}$$

G.8.3 All Equilibrium Conditions

The linearized competitive equilibrium consists of 13 endogenous allocations $\{\hat{C}_t,\ \hat{C}^e_t,\ \hat{I}_t,\ \hat{Y}_t, \hat{\kappa}_t,\ \hat{\bar{\omega}}_t,\ \hat{N}_t,\ \hat{K}_t,\ \hat{H}_t, \hat{l}_t,\ \hat{f}_t,\ \hat{d}_t,\ \hat{T}_t\}$, 15 prices $\{\hat{w}_t,\ \hat{w}^e_t,\ \hat{\pi}_t,\ \hat{R}_t,\ \hat{R}^{crp}_t,\ \hat{R}^k_t,\ \hat{R}^r_t,\ \hat{R}^e_t,\ \hat{s}_t, \hat{X}_t,\ \hat{Q}_t,\ \hat{\Phi}_t,\ \hat{R}^{new}_t,\ \hat{R}^{ave}_t,\ \hat{R}^{mp}_t\}$, and 4 exogenous processes $\{\hat{G}_t,\ \hat{\sigma}_{\omega,t},\ \hat{A}_t,\ \hat{\nu}^{\Phi}_t\}$; such that households, primary market participants, final good producers, retailers, wholesalers, capital producers, and entrepreneurs optimize, the central bank follows a Taylore rule, the treasury follows the tax rule, and markets clear. The equilibrium is characterized by the following equations and processes:

$$
\begin{aligned}
-\sigma\hat{C}_t + \hat{w}_t &= \eta\hat{H}_t \\
-\sigma\hat{C}_t &= -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \frac{1}{1+R}\hat{R}_t \\
-\sigma\hat{C}_t &= -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \frac{1}{1+R^{mp}}\hat{R}^{mp}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] \\
\hat{\pi}_t &= -\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{X}_t + \beta\mathbb{E}_t\left(\hat{\pi}_{t+1}\right) \\
\hat{Y}_t &= \hat{A}_t + \alpha\hat{K}_{t-1} + (1-\alpha)\Omega\hat{H}_t \\
\hat{R}^r_t &= -\hat{X}_t + \hat{Y}_t - \hat{K}_{t-1} \\
\hat{w}_t &= -\hat{X}_t + \hat{Y}_t - \hat{H}_t \\
\hat{w}^e_t &= -\hat{X}_t + \hat{Y}_t \\
\hat{K}_t &= \delta\hat{I}_t + (1-\delta)\hat{K}_{t-1} \\
\hat{Q}_t &= \delta\phi_K(\hat{I}_t - \hat{K}_{t-1}) \\
\hat{R}^k_{t+1} &= R^r\hat{R}^r_{t+1} + (1-\delta)\hat{Q}_{t+1} - (R^r + (1-\delta))\hat{Q}_t \\
\hat{\kappa}_t &= \hat{Q}_t + \hat{K}_t - \hat{N}_t \\
\frac{\mathbb{E}_t\hat{s}_{t+1}}{1+s} &= \psi_{s,\kappa}\hat{\kappa}_t + \psi_{s,\sigma_\omega}\hat{\sigma}_{\omega,t} \\
\hat{\omega}_{t+1} &= \psi_{\omega,\kappa}\hat{\kappa}_t + \psi_{\omega,s}\frac{\hat{s}_{t+1}}{1+s} + \psi_{\omega,\sigma_\omega}\hat{\sigma}_{\omega,t} \\
\hat{R}^e_t &= \kappa\left(\hat{R}^k_t - \hat{R}^{crp}_{t-1}\right) + \kappa(1+R^{crp})(s)\hat{\kappa}_{t-1} + \hat{R}^{crp}_{t-1} \\
&\quad - \mu G(\bar{\omega},\sigma_\omega)(1+R^{crp})(1+s)\kappa\left(\frac{\hat{R}^k_t}{1+R^k} + \hat{\kappa}_{t-1} + \frac{G_\omega\bar{\omega}}{G(\bar{\omega},\sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma\sigma_\omega}{G(\bar{\omega},\sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right)
\end{aligned}
$$

$$\hat{N}_t = \gamma(1+R^e)\left(\frac{\hat{R}^e_t}{1+R^e} + \hat{N}_{t-1}\right) + \frac{(1-\alpha)(1-\Omega)}{\alpha} R^r \kappa \hat{w}^e_t$$

$$\hat{C}^e_t = \hat{N}_{t-1} + \frac{\hat{R}^e_t}{1+R^e}$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{C^e}{Y}\hat{C}^e_t + \frac{I}{Y}\hat{I}_t + \bar{G}\hat{G}_t$$

$$+ \mu G(\bar{\omega}, \sigma_\omega)(1+R^k)\frac{K}{Y}\left(\hat{\kappa}_{t-1} + \hat{N}_{t-1} + \frac{\hat{R}^k_t}{1+R^k} + \frac{G_\omega \bar{\omega}}{G(\bar{\omega}, \sigma_\omega)}\hat{\omega}_t + \frac{G_\sigma \sigma_\omega}{G(\bar{\omega}, \sigma_\omega)}\hat{\sigma}_{\omega,t-1}\right)$$

$$\frac{1}{1+s}\hat{s}_t = \frac{1}{1+R^k}\hat{R}^k_t - \frac{1}{1+R^{crp}}\hat{R}^{crp}_{t-1}$$

$$\frac{1}{1+R^{crp}}\hat{R}^{crp}_t = \frac{1}{1+R}\hat{R}_t + \frac{1}{1+\Phi}\hat{\Phi}_t$$

$$\hat{\Phi}_t = \zeta\left(\hat{b}^{crp}_t + \hat{l}_t\right) + \nu^\Phi_t$$

$$-\frac{1}{1+\Phi}\hat{\Phi}_t + \frac{1}{(\delta^d + R^{new})}\hat{R}^{new}_t = \frac{1}{1+R^{mp}}\hat{R}^{mp}_t + (1-\delta^d)\beta\mathbb{E}_t\left[-\frac{1}{1+\Phi}\hat{\Phi}_{t+1} + \frac{1}{(\delta^d + R^{new})}\hat{R}^{new}_{t+1}\right]$$

$$\hat{n}_t = \bar{d}\hat{R}^{ave}_{t-1} + (R^{new} + \delta^d)\hat{d}_{t-1} - (R^{new} + \delta^d)\bar{d}\hat{\pi}_t$$

$$\hat{d}_t = (1-\delta^d)\hat{d}_{t-1} - (1-\delta^d)\bar{d}\hat{\pi}_t + \hat{l}_t$$

$$\hat{R}^{ave}_t = \left(1-\delta^d\right)\hat{R}^{ave}_{t-1} + \delta^d \hat{R}^{new}_t$$

$$\hat{n}_t = (R^{new}\bar{d} + \bar{G})\hat{T}_t - \bar{G}\hat{G}_t + \hat{l}_t$$

$$(R^{new}\bar{d} + \bar{G})\hat{T}_t = \bar{G}\hat{G}_t + \tau_T R^{new}\hat{d}_{t-1}$$

$$\hat{R}^{mp}_t = \rho^{mp}\hat{R}^{mp}_{t-1} + (1-\rho^{mp})[\phi_\pi \mathbb{E}_t \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t] + \varepsilon^{mp}_t$$

$$\hat{G}_t = \rho^G \hat{G}_{t-1} + \varepsilon^G_t$$

$$\hat{A}_t = \rho^A \hat{A}_{t-1} + \varepsilon^A_t$$

$$\hat{\sigma}_{\omega,t} = \rho^{\sigma_\omega}\hat{\sigma}_{\omega,t-1} + \varepsilon^{\sigma_\omega}_t$$

$$\hat{\nu}^\Phi_t = \rho^\Phi \hat{\nu}^\Phi_{t-1} + \varepsilon^\Phi_t$$

G.9 Formulae for Entrepreneurs Helping Functions

In this section, I present all the derivatives of the entrepreneurs functions $\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ and $G(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ which are used in the log-linearized equilibrium. The derivations are available upon request. Let me start with the definition of these two functions, the CFD, and the PFD of the log-normal distribution:

$$\Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega + \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t}))$$
$$G(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t}) d\omega$$
$$F(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \equiv \int_0^{\bar{\omega}_{t+1}} f(\omega, \sigma_{\omega,t}) d\omega$$
$$f(\omega, \sigma_{\omega,t}) \equiv \frac{1}{\omega \sigma_{\omega,t}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(\omega) + \frac{\sigma_{\omega,t}^2}{2}}{\sigma_{\omega,t}}\right)^2\right)$$

The first two derivatives, are the first derivatives of the PDF (f) with respect to ω and σ:

$$\frac{\partial f(\omega, \sigma_{\omega,t})}{\partial \omega} \equiv f_\omega(\omega, \sigma_{\omega,t}) = -\frac{1}{\omega} f(\omega, \sigma_{\omega,t}) \left(\frac{\ln(\omega)}{\sigma_{\omega,t}^2} + \frac{3}{2}\right)$$
$$\frac{\partial f(\omega, \sigma_{\omega,t})}{\partial \sigma_{\omega,t}} \equiv f_\sigma(\omega, \sigma_{\omega,t}) = -\frac{1}{\sigma_{\omega,t}} f(\omega, \sigma_{\omega,t}) + \frac{1}{\sigma_{\omega,t}} f(\omega, \sigma_{\omega,t}) \left(\frac{\ln(\omega)^2}{\sigma_{\omega,t}^2} - \frac{\sigma_{\omega,t}^2}{4}\right)$$

The derivatives of the CFD with respect to the threshold $\bar{\omega}_{t+1}$ and the variance $\sigma_{\omega,t}$, there ϕ is the PDF of a standard normal:

$$\frac{\partial F(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \bar{\omega}_{t+1}} \equiv F_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = f(\bar{\omega}_{t+1}, \sigma_{\omega,t})$$
$$\frac{\partial F(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \sigma_{\omega,t}} \equiv F_\sigma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma_{\omega,t}^2} - \frac{1}{2}\right) \bar{\omega}_{t+1} \sigma_{\omega,t} f(\bar{\omega}_{t+1}, \sigma_{\omega,t})$$

The first derivatives of G with respect to the threshold $\bar{\omega}_{t+1}$ and the variance $\sigma_{\omega,t}$:

$$\frac{\partial G(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \bar{\omega}_{t+1}} \equiv G_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \bar{\omega}_{t+1} f(\bar{\omega}_{t+1}, \sigma_{\omega,t})$$
$$\frac{\partial G(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \sigma_{\omega,t}} \equiv G_\sigma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = -\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma_{\omega,t}^2} + \frac{1}{2}\right) \bar{\omega}_{t+1}^2 \sigma_{\omega,t} f(\bar{\omega}_{t+1}, \sigma_{\omega,t})$$

Second, the first derivatives of Γ with respect to the threshold $\bar{\omega}_{t+1}$ and the variance $\sigma_{\omega,t}$:

$$\frac{\partial \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \bar{\omega}_{t+1}} \equiv \Gamma_\omega(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = 1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})$$
$$\frac{\partial \Gamma(\bar{\omega}_{t+1}, \sigma_{\omega,t})}{\partial \sigma_{\omega,t}} \equiv \Gamma_\sigma(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = -\bar{\omega}_{t+1}^2 \sigma_{\omega,t} f(\bar{\omega}_{t+1}, \sigma_{\omega,t})$$

The second derivatives of G starting from G_ω:

$$\frac{\partial^2 G(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\bar{\omega}_{t+1}\partial\bar{\omega}_{t+1}} \equiv G_{\omega\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t}) = -f(\bar{\omega}_{t+1},\sigma_{\omega,t})\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}}+\frac{1}{2}\right)$$
$$\frac{\partial^2 G(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\bar{\omega}_{t+1}\partial\sigma_{\omega,t}} \equiv G_{\omega\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) = \frac{1}{\sigma_{\omega,t}}\bar{\omega}_{t+1}f(\omega,\sigma_{\omega,t})\left(-1+\frac{\ln(\omega)^2}{\sigma^2_{\omega,t}}-\frac{\sigma^2_{\omega,t}}{4}\right)$$

Now starting from G_σ for G_σ:

$$\begin{aligned}\frac{\partial^2 G(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\sigma_{\omega,t}\partial\sigma_{\omega,t}} &\equiv G_{\sigma\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t})\\ &= 2\frac{\ln(\bar{\omega}_{t+1})}{\sigma^3_{\omega,t}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\bar{\omega}_{t+1})-\frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right]\\ &-\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}}+\frac{1}{2}\right)^2\left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}}-\frac{1}{2}\right)\frac{1}{\sigma_{\omega,t}}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\bar{\omega}_{t+1})-\frac{\sigma^2_{\omega,t}}{2}}{\sigma_{\omega,t}}\right)^2\right]\end{aligned}$$

The second derivatives of Γ starting from Γ_ω:

$$\frac{\partial^2 \Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\bar{\omega}_{t+1}\partial\bar{\omega}_{t+1}} \equiv \Gamma_{\omega\omega}(\bar{\omega}_{t+1},\sigma_{\omega,t}) = -f(\bar{\omega}_{t+1},\sigma_{\omega,t})$$
$$\frac{\partial^2 \Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\bar{\omega}_{t+1}\partial\sigma_{\omega,t}} \equiv \Gamma_{\omega\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) = \left(\frac{\ln(\bar{\omega}_{t+1})}{\sigma^2_{\omega,t}}-\frac{1}{2}\right)\bar{\omega}_{t+1}\sigma_{\omega,t}f(\bar{\omega}_{t+1},\sigma_{\omega,t})$$

Now from Γ_σ for $\Gamma_{\sigma\sigma}$:

$$\frac{\partial^2 \Gamma(\bar{\omega}_{t+1},\sigma_{\omega,t})}{\partial\sigma_{\omega,t}\partial\sigma_{\omega,t}} \equiv \Gamma_{\sigma\sigma}(\bar{\omega}_{t+1},\sigma_{\omega,t}) = -\bar{\omega}^2_{t+1}f(\omega,\sigma_{\omega,t})\left(\frac{\ln(\omega)^2}{\sigma^2_{\omega,t}}-\frac{\sigma^2_{\omega,t}}{4}\right)$$

H Further Model Results

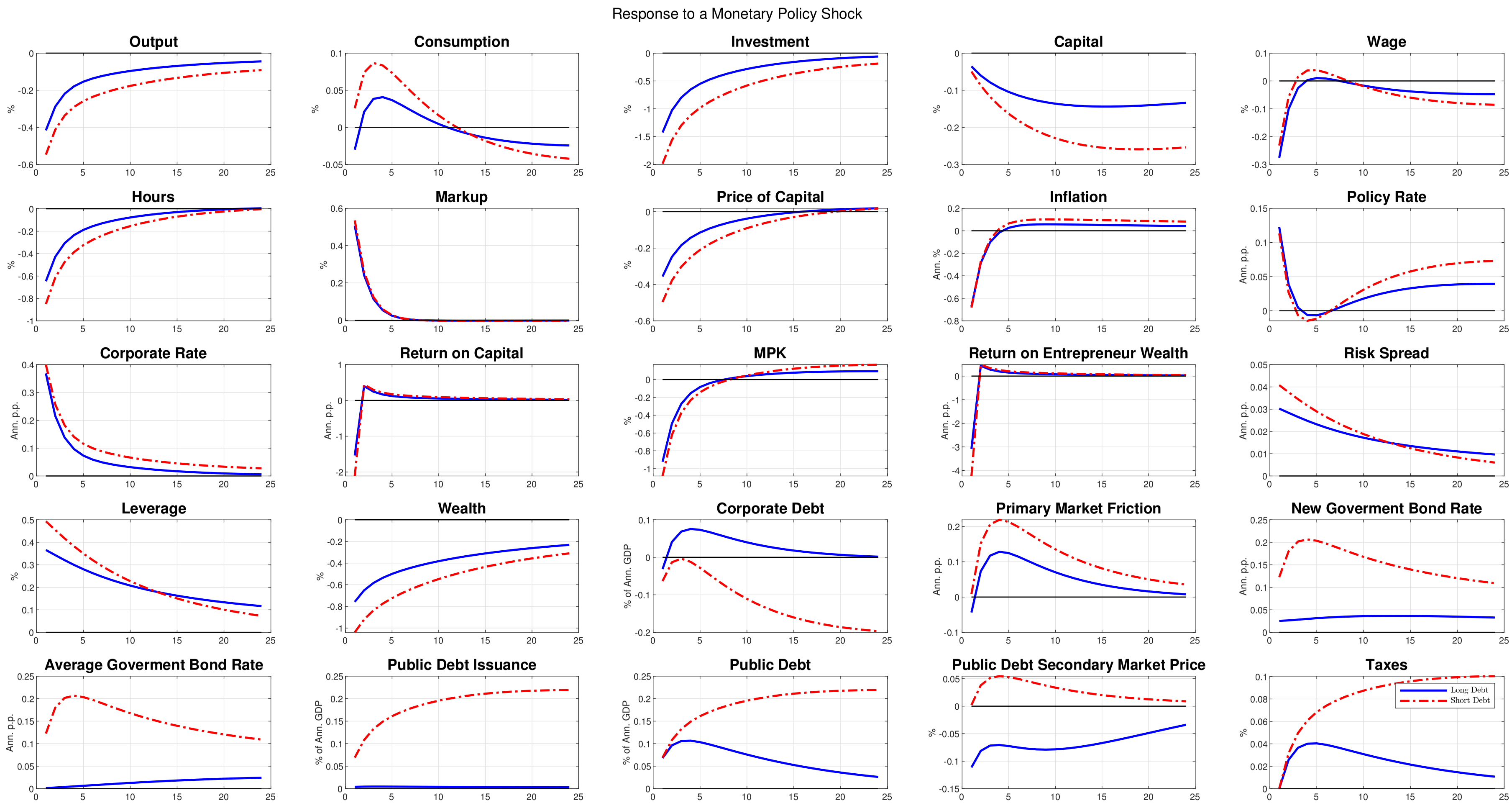
H.1 All Impulse Response Functions

Figure H.1 presents the whole set of impulse response functions to a contractionary 25 basis points monetary policy shocks in the baseline model, it is the more complete version of Figure 6. We can see the responses with public debt at its historical duration of around 4 years on the solid blue line and the responses in a counterfactual world with only one period public debt with the red dash-dot line.

In the baseline long debt case, output declines by about 40 basis points and investment by 1.4 percent, inflation by 0.6 percent in annualized terms. Leverage increases by 0.4 percent and the risk spread by 3 basis points. These are all standard results for a financial accelerator model and are the ones discussed in the main text in Section 7.2. On the additional IRFs a few stand out, first of all, the consumption response is muted, being an order of magnitude lower than output and investment. At most consumption declines by 3 basis points. This is in line with the empirical results. The interest rate in newly issued government debt increases only mildly. The reason is that, this is a long rate and the monetary policy shock is temporary in nature. Public debt jumps up in real terms as inflation declines have a Fisherian effect of increasing the debt burden. On the other hand, the secondary market value of public debt declines, as the existing debt rate as a lower average rate than the newly issued debt. This effect is not overturned by the higher primary market friction, which pushes upward the value of public debt. Taxes adjust slowly to respond to higher funding costs. The remaining part of the IRFs are standard for a financial accelerator New Keynesian model. Capital, wages, hours worked, the price of capital, the ex-post return on capital, the marginal productivity of capital, the ex-post return on entrepreneurs wealth and entrepreneurs wealth all decline on impact following a contractionary monetary policy shock. On the other hand markups increase.

If we turn to the responses in the one period debt presented with the dot-dashed red line, we can see how the government must increase issuance as the average rate on public debt shoots up. As discussed in Section 7.2, the primary market friction increases and this creates a further amplification mechanism with the financial accelerator. Output and investment decline relatively more, and leverage and the risk spread increase relatively more. A few points come from the additional set of IRFs presented here. First of all, consumption still does not respond much, it is even positive on impact, but its magnitude is low at 3 basis points. Interestingly, the policy rate responds relatively less in this scenario, but the difference is very minor, as in the data. If we analyze public debt, we can see that the initial response is the same as in the long-debt case due to the Fisherian effect of inflation on nominal debt. On the other hand, in the following periods public debt keeps rising due to the higher issuance costs. By the same token taxes keep rising to avoid an explosive path for public debt. Finally, we can see how the secondary market price of public debt jumps due to the higher friction; in this case, the primary market price of public debt is always one as the coupons adjust to ensure it.

Figure H.1: Baseline Model Impulse Response Functions For All Variables



Notes: The IRFs present the response to an annualized 25 basis points monetary policy shock. The solid blue line presents the IRFs in an economy with the maturity of public debt being at its historical average of around 4 years ($\delta^d = 0.05$). The dot-dashed red line presents the IRFs in an alternative economy with the maturity of public debt being at one quarter ($\delta^d = 1$).

Table H.1: Parameters Varied in Sensitivity Analysis

Parameter	Mean	Standard Deviation	Distribution
κ	2	1	Shifted Gamma (from 1)
s	0.0025	0.0025	Beta
$F(\omega, \sigma_\omega)$	0.0075	0.0075	Beta
ϕ_π	1.5	0.1	Normal
ϕ_Y	0.125	0.03	Normal
ρ^{mp}	0.8	0.05	Beta
θ	0.65	0.05	Beta
ζ	0.1	0.05	Beta
Φ	0.0025	0.0025	Beta
σ	2	0.5	Gamma
τ_T	2	0.5	Gamma
$\bar{D}$	1.6	0.5	Gamma

Notes: The first column shows which parameter is being varies. The second column shows the mean of the chosen distribution, this is equal to the baseline model calibration presented in Table 2. The third column shows the standard deviation of the sensitivity distribution. Finally, the fourth column specifies the distribution the parameter is being drawn from. With respect to leverage κ, the draws come from a shifted Gamma. That is, I draw from a Gamma with mean 1 and standard deviation 1 and then add 1 to each draw. The reason is that, in this model, leverage can go from 1 to infinity, and the shifted distribution allows for this while being centered at its calibrated value 2.

H.2 Sensitivity

It is important to check that the results do not hinge on any specific calibration, so in this section I perform a sensitivity analysis similar in spirit to a prior predictive analysis. The distribution of a few selected parameters are presented in table H.1. In this table, I use the posterior percentiles from either Smets and Wouters (2007) or Herbst and Schorfheide (2015) for ρ^{mp}, θ, and ϕ_Y to inform their distributions.

In Table H.2, we can see the outcome of this sensitivity analysis. In the first set of results, I show how some key metrics vary when all the parameters discussed above change, in the subsequent ones, I let one parameter vary at the time. For each case, I present a comparison between the high debt maturity case and the low debt maturity case. The first metric chosen is the percent difference in monetary policy strength on impact, how much more strongly output responds in a low maturity world. The other metrics show the highest difference (in absolute value) for a number of variables, all annualized.

The result is that leverage κ has an impact on the difference between high and low maturity on leverage and wealth, risk spread s only on spreads, and default rate $F(\omega, \sigma_\omega)$ as well only on spreads. The steady state value for the primary market friction Φ does not have a big impact, reassuringly. The tax parameter τ_T only has an impact on taxes, but not on much else. The steady state level of debt $\bar{D}$ has an impact on the difference between high and low maturity on corporate debt, output, the primary market friction, investment, wealth, and taxes. This is consistent with the idea that the impact of maturity of public debt matters when it can insure a big amount of debt to interest rate changes (it matters more for a high debt country as Japan than for a low debt one as Luxembourg). Interestingly, it matters mainly for the low maturity world, consistent with the idea that the primary market friction would hit more if all debt must be renewed each period. On the other hand, in the baseline model with high maturity the only thing that moves with debt are taxes and debt itself. The primary market friction elasticity ζ has a big impact on the difference of output, corporate debt,the primary market friction, investment, wealth, and leverage. However, on absolute does not matter much as we can see in the sensitivity on high maturity case. Most results are again driven by changes in the low maturity case. The parameters relating to the Taylor rule and price stickiness can have a large impact on the absolute results as in a standard small size New-Keynesian model and this one is no different, however, in general the relative magnitudes of my finding remain unchanged and they do not hit particularly the primary market friction, that is no parameter kills it.

When we draw from all parameter distributions together we obtain a 90% confidence interval for how much more effective would have been monetary policy on output under a short debt scenario that ranges from 8% to 92% more effective, that is in basis points a difference that goes from 4 to 72 basis points difference following a 25 basis points increase in interest rates. Also in this case, all relative statements go through, with the most variation seen in the response of investment, corporate debt, public debt but not much for inflation and the policy rate. When drawing for each parameter alone, I did 500 draws, when all parameters together, 50000.

H.3 Sensitivity of the Maturity Structure

In Section 7.4 we could see how maturity is the key to find the quantitative results on the strength of monetary policy and that changes in steady state public debt would have needed to be too high to deliver the same quantitative result. Therefore, a natural question that comes is how the results vary with the maturity structure. In the baseline experiment, I compared a one period (quarter) debt with debt duration of about 4 years ($\delta^d = 0.05$), so that 5% of the debt needs to be refinanced every quarter, in line with historical averages for the United States. In order to answer this question I plot in Figure H.2 the peak response of output to a monetary policy shock for different levels of δ^d. I allow δ^d to vary from 1 (1 quarter) to 0.0066 (15 years). The figure clearly shows a linear relationship, with more debt needed to be refinanced leading to stronger monetary policy responses. It is worth noting that Macaulay duration has an hyperbola-like formula $\frac{1+R^{new}}{\delta^d+R^{new}}$, so that increasing debt duration by one year from 1 quarter debt will have a stronger effect on the output response than from 10 years.

H.4 Equal Monetary Policy Rate Response Across Regimes

In the baseline exercise, I study the effects of a 25 basis annualized monetary policy shock in both the long and short maturity regimes. However, this implies a different observed Monetary Policy Rate R_t^{mp} on impact due to the endogenous response of output and inflation and their

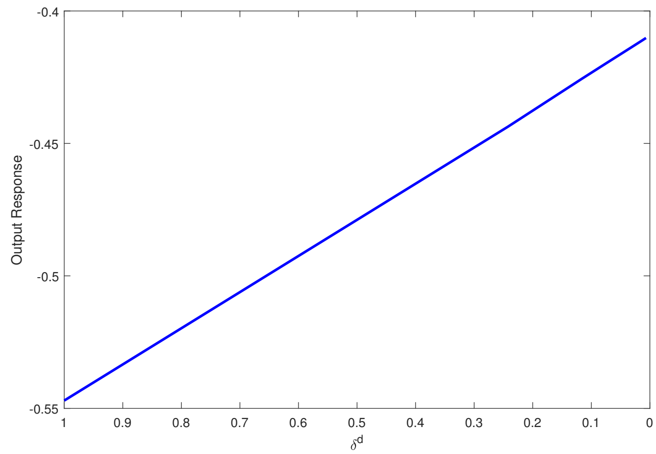
Table H.2: Sensitivity Analysis for Difference in Response to a Monetary Policy Shock under High and Low Maturity Models

	Metric	Mean	Median	5th Percentile	95th Percentile
All Parameters	Percent Difference in Output	-0.3684	-0.2735	-0.9145	-0.0778
	Difference in Output	0.1728	0.1064	0.0330	0.4551
	Difference in Risk Spread	-0.0209	-0.0029	-0.0576	0.0000
	Difference in Primary Market Friction	-0.1150	-0.0775	-0.2897	-0.0240
	Difference in Investment	0.7182	0.4482	0.1431	1.8094
	Difference in Corporate Debt	0.2101	0.1688	0.0637	0.4865
	Difference in Public Debt Issuance	-0.2260	-0.1867	-0.5178	-0.0778
	Difference in Inflation	0.1249	0.0203	-0.0397	0.4256
κ	Percent Difference in Output	-0.3577	-0.3379	-0.4711	-0.2894
	Difference in Output	0.1409	0.1386	0.1296	0.1570
	Difference in Risk Spread	-0.0114	-0.0090	-0.0293	-0.0023
	Difference in Primary Market Friction	-0.1003	-0.0972	-0.1240	-0.0851
	Difference in Investment	0.5918	0.5864	0.5582	0.6630
	Difference in Corporate Debt	0.2008	0.2041	0.1838	0.2101
	Difference in Public Debt Issuance	-0.2162	-0.2187	-0.2255	-0.2019
	Difference in Inflation	0.0339	0.0243	0.0031	0.0875
s	Percent Difference in Output	-0.3061	-0.3010	-0.3498	-0.2775
	Difference in Output	0.1263	0.1231	0.1098	0.1524
	Difference in Risk Spread	-0.0091	-0.0063	-0.0269	-0.0005
	Difference in Primary Market Friction	-0.0877	-0.0858	-0.1035	-0.0773
	Difference in Investment	0.5408	0.5243	0.4624	0.6703
	Difference in Corporate Debt	0.2039	0.2046	0.1929	0.2128
	Difference in Public Debt Issuance	-0.2174	-0.2176	-0.2224	-0.2117
	Difference in Inflation	0.0057	0.0023	-0.0098	0.0315
$F(\omega, \sigma_\omega)$	Percent Difference in Output	-0.3199	-0.3178	-0.3430	-0.3036
	Difference in Output	0.1350	0.1334	0.1235	0.1518
	Difference in Risk Spread	-0.0129	-0.0118	-0.0220	-0.0071
	Difference in Primary Market Friction	-0.0926	-0.0919	-0.1007	-0.0869
	Difference in Investment	0.5768	0.5706	0.5331	0.6402
	Difference in Corporate Debt	0.1993	0.1997	0.1923	0.2052
	Difference in Public Debt Issuance	-0.2147	-0.2149	-0.2186	-0.2102
	Difference in Inflation	0.0139	0.0122	0.0024	0.0310
ζ	Percent Difference in Output	-0.3005	-0.3042	-0.4258	-0.1754
	Difference in Output	0.1238	0.1258	0.0633	0.1835
	Difference in Risk Spread	-0.0100	-0.0101	-0.0147	-0.0052
	Difference in Primary Market Friction	-0.0856	-0.0871	-0.1268	-0.0435
	Difference in Investment	0.5310	0.5398	0.2766	0.7797
	Difference in Corporate Debt	0.1912	0.1965	0.1297	0.2403
	Difference in Public Debt Issuance	-0.2079	-0.2119	-0.2495	-0.1567
	Difference in Inflation	0.0088	0.0081	-0.0064	0.0283
Φ	Percent Difference in Output	-0.3141	-0.3111	-0.3324	-0.3048
	Difference in Output	0.1308	0.1308	0.1307	0.1309
	Difference in Risk Spread	-0.0105	-0.0104	-0.0113	-0.0102
	Difference in Primary Market Friction	-0.0907	-0.0896	-0.0970	-0.0875
	Difference in Investment	0.5606	0.5541	0.5398	0.6008
	Difference in Corporate Debt	0.2012	0.2027	0.1915	0.2065
	Difference in Public Debt Issuance	-0.2159	-0.2167	-0.2189	-0.2109
	Difference in Inflation	0.0096	0.0081	0.0050	0.0188
$\bar{D}$	Percent Difference in Output	-0.3035	-0.3017	-0.4337	-0.1740
	Difference in Output	0.1264	0.1255	0.0719	0.1819
	Difference in Risk Spread	-0.0102	-0.0101	-0.0147	-0.0058
	Difference in Primary Market Friction	-0.0875	-0.0870	-0.1247	-0.0504
	Difference in Investment	0.5428	0.5376	0.3053	0.7879
	Difference in Corporate Debt	0.1999	0.1895	0.0918	0.3401
	Difference in Public Debt Issuance	-0.2142	-0.2038	-0.3585	-0.1011
	Difference in Inflation	0.0078	0.0103	-0.0051	0.0125
τ_T	Percent Difference in Output	-0.3179	-0.3162	-0.3376	-0.3050
	Difference in Output	0.1323	0.1316	0.1271	0.1403
	Difference in Risk Spread	-0.0107	-0.0106	-0.0112	-0.0103
	Difference in Primary Market Friction	-0.0918	-0.0913	-0.0982	-0.0878
	Difference in Investment	0.5661	0.5637	0.5477	0.5938
	Difference in Corporate Debt	0.1959	0.1968	0.1633	0.2247
	Difference in Public Debt Issuance	-0.2123	-0.2123	-0.2364	-0.1875
	Difference in Inflation	0.0128	0.0113	0.0021	0.0292

	Metric	Mean	Median	5th Percentile	95th Percentile
σ	Percent Difference in Output	-0.3094	-0.3120	-0.3487	-0.2604
	Difference in Output	0.1290	0.1300	0.1104	0.1441
	Difference in Risk Spread	-0.0105	-0.0105	-0.0111	-0.0098
	Difference in Primary Market Friction	-0.0900	-0.0903	-0.0954	-0.0834
	Difference in Investment	0.5579	0.5594	0.5266	0.5837
	Difference in Corporate Debt	0.2007	0.2008	0.1972	0.2039
	Difference in Public Debt Issuance	-0.2154	-0.2156	-0.2187	-0.2117
	Difference in Inflation	0.0084	0.0091	-0.0037	0.0181
θ	Percent Difference in Output	-0.3276	-0.3178	-0.4936	-0.1898
	Difference in Output	0.1374	0.1324	0.0772	0.2126
	Difference in Risk Spread	-0.0111	-0.0107	-0.0170	-0.0062
	Difference in Primary Market Friction	-0.0956	-0.0918	-0.1507	-0.0531
	Difference in Investment	0.5879	0.5677	0.3349	0.8995
	Difference in Corporate Debt	0.2063	0.2024	0.1537	0.2699
	Difference in Public Debt Issuance	-0.2214	-0.2173	-0.2902	-0.1647
	Difference in Inflation	0.0203	0.0107	-0.0147	0.0867
ϕ_π	Percent Difference in Output	-0.3052	-0.3129	-0.3592	-0.2279
	Difference in Output	0.1266	0.1303	0.0961	0.1459
	Difference in Risk Spread	-0.0102	-0.0105	-0.0117	-0.0078
	Difference in Primary Market Friction	-0.0884	-0.0904	-0.0968	-0.0739
	Difference in Investment	0.5440	0.5591	0.4237	0.6151
	Difference in Corporate Debt	0.1982	0.2008	0.1840	0.2040
	Difference in Public Debt Issuance	-0.2127	-0.2155	-0.2190	-0.1973
	Difference in Inflation	0.0052	0.0088	-0.0419	0.0395
ϕ_Y	Percent Difference in Output	-0.3532	-0.3177	-0.6281	-0.1768
	Difference in Output	0.1617	0.1328	0.0664	0.3377
	Difference in Risk Spread	-0.0128	-0.0107	-0.0263	-0.0051
	Difference in Primary Market Friction	-0.1039	-0.0916	-0.1881	-0.0537
	Difference in Investment	0.6743	0.5687	0.2982	1.3457
	Difference in Corporate Debt	0.2169	0.2025	0.1446	0.3292
	Difference in Public Debt Issuance	-0.2328	-0.2173	-0.3536	-0.1551
	Difference in Inflation	0.0566	0.0119	-0.0313	0.2751
ρ^{mp}	Percent Difference in Output	-0.3060	-0.3120	-0.3156	-0.2739
	Difference in Output	0.1330	0.1330	0.0942	0.1727
	Difference in Risk Spread	-0.0108	-0.0107	-0.0141	-0.0076
	Difference in Primary Market Friction	-0.0934	-0.0922	-0.1256	-0.0664
	Difference in Investment	0.5735	0.5705	0.4045	0.7573
	Difference in Corporate Debt	0.2101	0.2048	0.1454	0.2961
	Difference in Public Debt Issuance	-0.2255	-0.2198	-0.3178	-0.1561
	Difference in Inflation	0.0025	0.0089	-0.0345	0.0125

Notes: The table shows a sensitivity analysis on the how high debt maturity case and the low debt maturity case respond differently following a 25 basis points contractionary monetary policy shock. The first column, shown sideways, display which scenario is being considered. In the first case (All Parameters), I allow all parameters to vary according to the distribution discussed in Table H.1. All other set of results show the same information when we allow to vary only one parameter at the time, where this parameter is shown sideways in the first column. In the first parameter is being varied. The second column shows the metric displayed. The remaining columns show for that metric, the average, the median, the 5th percentile, and the 95th percentiles of the draws that produced a model solution. In each scenario, I present the eight metrics, all pertaining to the different behavior under the two maturity cases. The first metric show is the percent difference in monetary policy strength on impact, how much more strongly output responds in a low maturity world. The other seven metrics show the highest difference (in absolute value) between the high debt maturity case and the low debt maturity case, for a number of variables, all annualized. As an example, a positive value for the difference in investment implies that investments response is relatively higher under the high maturity case than under the low maturity case. The first experiment with all parameters has 50000 draws, the other experiments, when I vary one parameter at the time has 500 draws. I keep all draws which for which both models (low and high maturity) admit a solution.

Figure H.2: Maximum Output Response Varying Public Debt Maturity



Notes: This figure presents the response on output on impact to an annualized 25 basis points monetary policy shock, for different values of the maturity structure δ^d. This parameter represents the fraction of the debt principal which must be refinanced each period, it goes on the horizontal axis from 1 (1 quarter) to 0.0066 (15 years). As we go to the right of the x-axis the maturity of public debt is increasing.

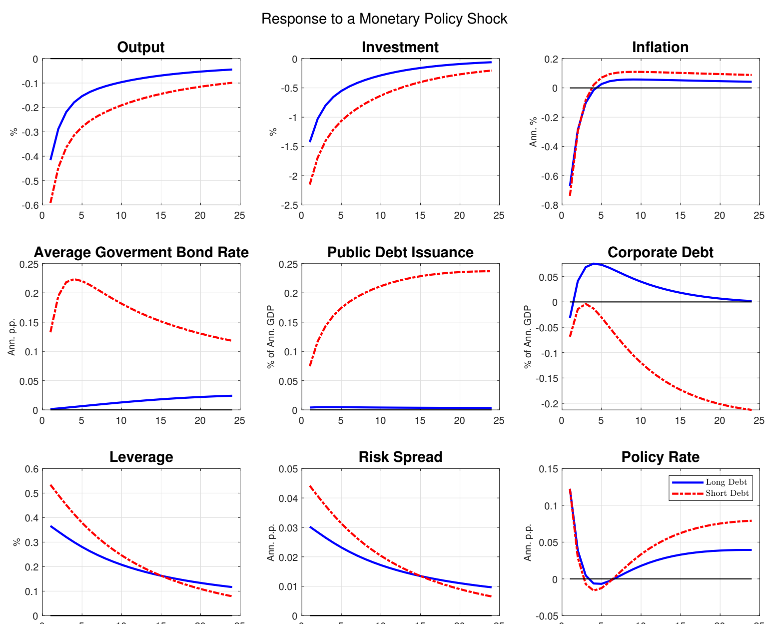
impact on the Taylor rule. We can see in the second row, fourth column of Figure H.1 that there is a small difference in responses of the policy rate on impact across regimes. On the other hand, the LP-IV empirical estimates condition on the monetary policy shock increasing the federal funds rate by the same amount across maturity regimes (one percent). Therefore, in this appendix, I apply the same conditioning on the model. For the short maturity regime, I pick the standard deviation of the monetary policy in order to observe the same response on impact of the policy rate as in the long maturity regime. The implementation is to multiply the standard deviation of the monetary policy shock in the low maturity regime by the ratio of the responses on impact of the policy rate in the average maturity regime over the short maturity regime: $s^{mp}|_{\delta^d \neq 1} \frac{\left.\frac{\partial R_t^{mp}}{\partial \varepsilon_t^{mp}}\right|_{\delta^d \neq 1}}{\left.\frac{\partial R_t^{mp}}{\partial \varepsilon_t^{mp}}\right|_{\delta^d = 1}}$.

We can see the results of this experiment in Figure H.3. In the last panel, we can see that the monetary policy rate reacts by the same amount in both regimes. The overall picture is very similar to Figure 6, with even stronger responses of output and investments under the short maturity regimes, and similar responses of inflation. The peak response difference in output increases from 13 to 18 basis points. The outcome of this exercise is to show that the results of the model are robust to studying the same structural shock magnitude across regimes or a structural shock with the same impact on a key endogenous variable across regimes.

H.5 Fisherian Channel for Corporate Debt

In the baseline model, I followed BGG and let the outside option for lenders to the entrepreneurs to be fixed in real terms. This would have been the optimal contract coming from trading

Figure H.3: Model Impulse Response Functions with Equal Monetary Policy Rate Response Across Regimes



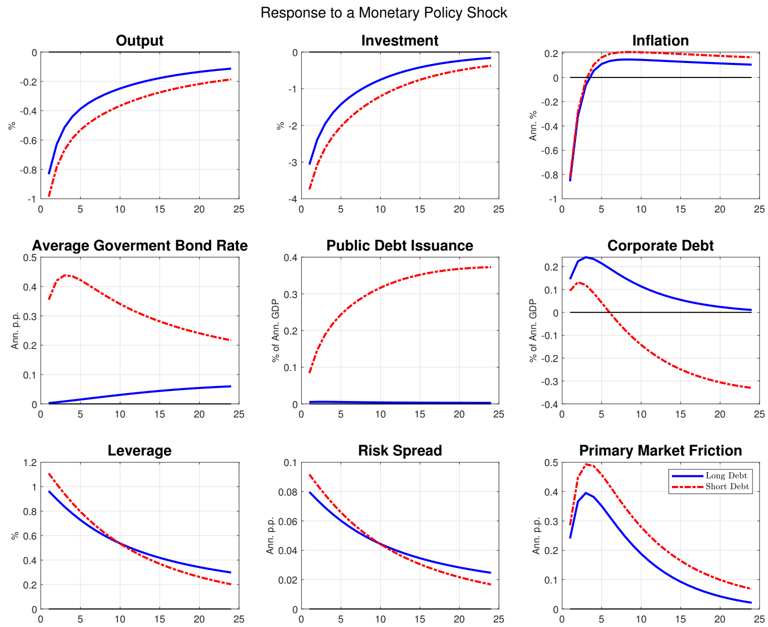
Notes: The IRFs present the response to a monetary policy shock. The solid blue line presents the IRFs in an economy with the maturity of public debt being at its historical average of around 4 years ($\delta^d = 0.05$). The dot-dashed red line presents the IRFs in an alternative economy with the maturity of public debt being at one quarter ($\delta^d = 1$). The blue line IRFs present the response to an annualized 25 basis points monetary policy shock and the red line to a monetary policy shock that increases the monetary policy rate on impact by the same amount.

between risk neutral entrepreneurs and risk averse households in the absence of labor risk. However, in presence of labor risk for risk averse households, the optimal contract would have entrepreneurs would insuring this risk by providing an indexed contract. This argument has been proposed by Carlstrom, Fuerst and Paustian (2016) and Dmitriev and Hoddenbagh (2017) among others. I kept that contract out of comparability with BGG and higher empirical relevance than a corporate bond with an interest rate with an amount of indexation not seen in the data. However, an even more realistic contract (even if even harder to argue in terms of optimal contracts with this set-up) would be one with the rate being fixed in nominal terms rather than real terms. This is the route proposed by Christiano, Motto and Rostagno (2014).

In the model, having a nominally fixed rate corporate debt at rate $R_t^{crp,nom}$, changes the Euler equation for corporate debt and the lenders participation constraints that entrepreneurs face when choosing leverage and default threshold. Specifically, the outside option is now risky ex-post as it varies with realized inflation, so that the realized risk spread is the ratio of the real return on capital investments over the realized real return on the nominal corporate debt: $1+s_{t+1} = \frac{1+R_{t+1}^k}{\frac{1+R_t^{crp,nom}}{\pi_{t+1}}}$. After solving the new equilibrium and taking the Taylor expansion around the zero inflation steady state, the changes are to substitute $\hat{R}_{t-1}^{crp}$ with $\hat{R}_{t-1}^{crp,nom} - \hat{\pi}_t$ and $\hat{R}_t^{crp}$ with $\hat{R}_t^{crp,nom} - \mathbb{E}(\hat{\pi}_{t+1})$ everywhere in the linearized equilibrium.

Figure H.4 presents the results from this experiment. Allowing for fixed nominal rate debt for corporate bonds creates a Fisherian debt deflation channel that makes the effects of monetary policy stronger. Following a contractionary monetary policy inflation declines, which increases the ex-post real return entrepreneurs must pay. The higher rate on liabilities lowers entrepreneurs wealth, thereby increasing leverage and the risk spread. This in turn lowers investments and output. By comparing the results with the baseline case of real debt from Figure 6, we can see how the monetary shock has stronger effects both under the high and low maturity scenarios. The peak effect on output, under the long maturity case, moves from -40 basis points in the real debt case to -80 basis points in the nominal debt case. If we move to the comparison across maturities, the difference is not really affected by the nature of corporate debt. The difference in output at impact between the long maturity case and the short maturity case increases from 13 basis points to 15, a negligible difference. The outcome of this experiment is to show how allowing for nominally fixed rate debt does not alter the conclusions on the differential effectiveness of monetary policy under the two maturity profiles.

Figure H.4: Model Impulse Response Functions with Nominal Fixed Rate Corporate Debt



Notes: The IRFs present the response to an annualized 25 basis points monetary policy shock. The solid blue line presents the IRFs in an economy with the maturity of public debt being at its historical average of around 4 years ($\delta^d = 0.05$). The dot-dashed red line presents the IRFs in an alternative economy with the maturity of public debt being at one quarter ($\delta^d = 1$). The corporate bond rate is specified as a one period debt with a fixed rate in nominal term.

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